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Exploring Student Mathematical Fluency in Understanding and Applying Basic Differential Calculus Rules: An Inquiry in Milton Margai Technical University - Sierra Leone

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***Abstract*—** This study looks at how well students understand and use the rules of differential calculus, a key part of mathematics. It finds that many students struggle with applying these rules correctly. Out of 120 university students enrolled for introduction to calculus course, about 69% perform poorly, while 8% are considered good and 4% are very good. The research shows that students' previous experience with calculus, the number of hours they study, and class attendance all help improve their understanding. Specifically, prior calculus experience has a moderate positive effect with a correlation of 0.45, study hours have a strong positive effect with a correlation of 0.52, and attendance has a moderate effect with a correlation of 0.30. On the other hand, feelings like confidence have a weak impact, with a correlation of only 0.12. Common problems include misunderstandings about concepts, procedural errors, and anxiety. 54.74% of the errors made by future teachers are due to conceptual misunderstandings, indicating a need for deeper understanding of ideas, not just calculation practice. The study recommends using teaching tools like visual aids, real-world examples, diverse exercises, and a supportive environment to improve learning. It also suggests curriculum changes and new teaching methods to help students become more fluent in calculus, which is essential for success in STEM fields.

***Keywords*—** Differential Calculus, Mathematical Fluency, Differential Rule, Diagnostic Assessments.

1. INTRODUCTION

Differential calculus is a fundamental branch of mathematics that forms the cornerstone of advanced mathematical understanding, particularly in engineering, physics, economics, and numerous STEM disciplines (Pelemeniano & Siega, 2023). The study of derivatives and their applications is essential for developing students' analytical thinking and problem-solving capabilities. Despite the vital role of calculus in advanced mathematics, students frequently struggle to develop proficiency in understanding and applying its basic differentiation rules. Many experience significant conceptual and procedural gaps that hinder their progression in higher-level mathematics, resulting in misconceptions and inadequate problem-solving skills (Rochaminah et al., 2025). This gap in understanding not only hampers their overall mathematical development but also affects their readiness for STEM careers. The challenge is exacerbated by the lack of comprehensive assessments of students' fluency across diverse contexts. Gaining insights into students' proficiency in applying differential calculus rules is therefore critical for designing targeted interventions that can improve learning outcomes.

The challenge of teaching and learning differential calculus extends beyond simple procedural fluency to encompass deep conceptual understanding. Traditional instructional approaches often emphasize computational techniques and algorithmic procedures without adequately addressing the fundamental concepts underlying differentiation (Prihandhika & Azizah, 2025). This disconnect between procedural knowledge and conceptual understanding has been a persistent concern in mathematics education, with students frequently able to execute derivative calculations without truly comprehending what those calculations represent or when they should be applied (Prihandika & Perbowo, 2024).

Mathematical fluency in calculus requires a sophisticated integration of multiple knowledge types and cognitive processes. Students must not only master the mechanical application of rules such as the power rule, product rule, quotient rule, and chain rule, but also develop mental representations that connect these rules to the graphical, numerical, and real-world interpretations of derivatives. The complexity of achieving this multifaceted understanding presents substantial pedagogical challenges that current educational systems continue to grapple with globally.

This study aims to evaluate students' proficiency in understanding and applying these differential calculus rules by assessing their conceptual grasp, measuring their problem-solving abilities, and identifying common misconceptions and difficulties they encounter.

By identifying specific areas of difficulty, educators can develop more effective instructional strategies and materials. Enhancing mathematical fluency is essential for fostering the analytical and problem-solving skills necessary for scientific and technological advancement (Lamon, 2001).

2. LITERATURE

2.0.1 Basic differential rule

The differential rule, also known as the derivative rule, is a fundamental concept in calculus used to find the derivative of a function. It provides a systematic way to differentiate functions based on their form (Bird, 2014).

Table 1: *Basic differential rules and derivative methods*

Rule	Differential Method
Constant	$\frac{d}{dx}[C] = 0$ where c is a constant
Power	$\frac{d}{dx}[x^n] = nx^{n-1}$ where n is any real number.
Constant Multiple	$\frac{d}{dx}[c.f(x)] = c.\frac{d}{dx}[f(x)]$
Sum	$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$
Difference	$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$
Product	$\frac{d}{dx}[f(x)\cdot g(x)] = \frac{d}{dx}[f(x)]\,[g(x)]+[f(x)]\frac{d}{dx}[g(x)]$
Quotient	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]\,[g(x)]-[f(x)]\frac{d}{dx}[g(x)]}{[g(x)]^2}$
Chain	$\frac{d}{dx}[f(g(x))] = \frac{d}{dx}(f(g(x))\cdot\frac{d}{dx}(g(x))$

2.0.2 The Nature of Mathematical Fluency in Calculus

Mathematical fluency involves both procedural skills and conceptual understanding, but research indicates these are often developed separately in calculus education (Rochaminah et al., 2025). Procedural knowledge pertains to executing algorithms and solving problems with established techniques, whereas conceptual knowledge involves grasping underlying principles and meanings. In differential calculus, procedural fluency means applying derivative rules efficiently, while conceptual understanding requires knowing why these rules work and what derivatives represent in different contexts. Studies of prospective teachers showed that although 82.5% could find derivatives mechanically, they lacked deeper conceptual insight, especially with complex tasks, highlighting that procedural skill alone doesn't ensure comprehension (Rochaminah et al., 2025). Students often struggle to demonstrate higher-order thinking and flexible application of procedures, emphasizing the need for understanding across multiple representations. Using the structure of observed learning outcomes (SOLO) taxonomy, research found that students with high conceptual understanding excelled across literacy indicators like communication, reasoning, and problem-solving; those with moderate or low proficiency showed limited skills, particularly in advanced strategies (Fernandez & Guzon, 2025). The mental concept image students' internal representations often diverges from formal definitions, especially for derivatives, where many students focus on symbolic and procedural aspects rather than integrating graphical and applied interpretations (Prihandika & Perbowo, 2024). This fragmented understanding creates epistemological obstacles that hinder solving conceptual problems and transferring knowledge. Assessing conceptual understanding effectively requires multidimensional approaches beyond traditional procedural tests. For example, in multivariable calculus, students could differentiate symbols but struggled to interpret gradient vectors or relate them to tangent planes (Andiani et al., 2025). Similarly, secondary students performed well on routine problems but faltered with more complex differentiation tasks, underscoring the difficulty of distinguishing deep understanding from memorized procedures. Diagnostic assessments incorporating open-ended, conceptual, and real-world tasks offer a more accurate picture of student comprehension than standard multiple-choice tests focused solely on procedural fluency.

2.0.3 Common Misconceptions and Errors in Differential Calculus

Research consistently highlights persistent misconceptions and systematic errors among students learning differentiation. Common issues include misunderstandings of the product rule, where students mistakenly believe the derivative of a product equals the product of derivatives, and difficulties with the chain rule, such as failing to recognize function composition or applying it incorrectly in nested cases. Logarithmic differentiation poses particular challenges, with 36.1% of pre-service teachers demonstrating misconceptions, often related to misapplication of rules or conceptual misunderstandings (Tatira & Mukuka, 2024). Despite repeated practice, these misconceptions often remain, indicating traditional drilling is insufficient. Error analysis reveals four main types: conceptual errors (misunderstanding derivative principles), procedural errors (incorrect application of steps), language errors (imprecise communication), and generalization errors (inappropriately extending procedures), with common mistakes involving exponent manipulation, fractions, and derivative calculations (Villavicencio, 2023). Studies on derivative-related curve properties show students often misinterpret positive derivatives as increasing intervals and negative derivatives as decreasing, reflecting internal logical systems inconsistent with mathematical theory (Chien et al., 2025). Difficulties in linking cubic graphs to their derivatives further illustrate underdeveloped understanding of the function-derivative relationship. Error patterns are consistent across various educational settings; pre-service teachers solving differential equations show procedural errors at 72.63%, computational errors at 67.37%, and conceptual errors at 54.74% (Meika & Sujana, 2026). In limit problems, foundational to understanding derivatives, conceptual errors constitute 67.6% of mistakes, underscoring that students' primary struggles are with conceptual understanding rather than procedural skills (Julaihi, 2025). These findings suggest instructional strategies should focus more on conceptual clarity rather than solely on practice, to effectively address and rectify these misconceptions.

2.0.4 Factors Affecting Student Understanding and Fluency

Students' ability to develop fluency in differential calculus is significantly influenced by both cognitive and affective factors. A lack of prerequisite knowledge in algebra, trigonometry, and foundational mathematical concepts creates barriers to understanding derivatives, often leading to errors and cognitive overload (Pelemeniano & Siega, 2023; Andiani et al., 2025). Additionally, individual differences such as learning styles, mathematical anxiety, and attitudes toward calculus play crucial roles; for instance, high anxiety can impede conceptual understanding and persistence, especially when students perceive calculus as abstract and disconnected from real-world relevance (Belza & Salcedo, 2025; Ajmera et al., 2025). The use of multiple representations algebraic, graphical, numerical, and contextual can enhance understanding, particularly when instruction emphasizes their integration, with dynamic visualizations further supporting conceptual development and transfer of knowledge (Prihandhika & Azizah, 2025; Kossivi, 2025). Ultimately, addressing foundational gaps, emotional factors, and employing effective visual tools are vital for fostering meaningful learning and overcoming the multifaceted challenges students face in mastering differential calculus.

2.0.5 Effective Teaching Strategies and Interventions for Developing Fluency

The research highlights the importance of using visual, intuitive, and contextualized teaching strategies to help students understand derivatives, emphasizing real-world applications like speed and growth ("An Overview of Strategies for Conceptualizing Derivative and Their Applications in Daily Life for Secondary-Level Mathematics Students," 2024). Employing multiple representations graphical, numeric, verbal, and symbolic supports robust conceptual development. Frameworks like the Hypothetical Learning Trajectory (HLT) guide instruction from intuitive ideas to formal definitions, with targeted activities addressing misconceptions and individual mental images enhancing learning (Prihandhika & Azizah, 2025; Tatira & Mukuka, 2024). Collaborative, problem-based, and inquiry-driven approaches, especially when connected to real-world problems such as geotechnical engineering, increase engagement, understanding, and performance, with students reporting stronger mental links to the material (Ajmera et al., 2025). The use of dynamic visualization tools like GeoGebra and Maple, along with interactive e-modules providing real-time feedback, greatly improves conceptual comprehension of derivatives and tangent planes, outperforming static visuals and allowing students to observe parameter effects (Lestari et al., 2024; Kossivi, 2025). Experiential, hands-on learning approaches that incorporate real-world problem-solving further enhance conceptual knowledge and reasoning, particularly for students struggling with abstraction. Overall, integrating technology, real-world contexts, and active engagement strategies fosters deeper understanding, motivation, and mastery of calculus concepts.

2.0.6 Challenges in Current Mathematics Education in Differential Calculus

The analysis of textbooks in calculus reveals that about 95% of worked examples and exercises lack real-world context and offer limited opportunities for modeling and mathematization, despite curriculum standards emphasizing the integration of practical applications (Nepal et al., 2024). This disconnect between curricular goals and textbook content hampers student understanding. Teachers often lack the training and confidence to incorporate real-world problem-solving approaches, facing obstacles such as curricular limitations, resource shortages, and inadequate professional development (Indirani, 2025). Systemic changes in teacher training and curriculum development are necessary to improve student learning. The cognitive challenges of calculus, coupled with students' gaps in prerequisite knowledge like algebra and trigonometry, lead to cognitive overload, making it difficult to master new content while remediating prior skills. These issues are prevalent globally, reflecting broader systemic challenges in early mathematics education. Time constraints, resource limitations, and diverse student preparedness further hinder the implementation of effective instructional strategies. Teachers using collaborative learning with technology also encounter challenges, including varied programming backgrounds among students, lack of formative assessments, and difficulties in differentiating instruction to meet diverse needs (Haarsa, 2025).

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3. METHODOLOGY

This study employed a mixed-method approach. Quantitative data was collected through diagnostic tests assessing students' ability to apply differential rules, while qualitative data was gathered via interviews and open-ended questionnaires to explore their conceptual understanding and misconceptions. Sample size for the study was 120 students enrolled in introductory calculus course at the Milton Margai Technical University. Data analysis involved descriptive statistics, correlation analysis, and thematic coding of qualitative responses.

4. RESULT

Table 2. *Diagnostic tests assessing students' ability to apply differential rules*

Score Range	Number of students	%	Performance level
0-39	57	48	Very poor
40-49	25	21	Poor
50-64	23	19	Average
65 - 74	10	8	Good
75 -100	5	4	Very good

Milton Margai Technical University score range values

The table displays the results of diagnostic assessments evaluating students' proficiency in applying differential rules. According to the data, 48% of students scored between 0 and 39, categorizing them as "Very Poor." 21% scored within the 40 to 49 range, placing them in the "Poor" category. Collectively, these two groups make up approximately 69% of the student population, indicating that the majority are experiencing significant difficulties with the application of differential rules.

Also, 19% of students scored between 50 and 64, which is considered "Average," reflecting moderate understanding among a smaller segment of the students. Only 8% of students scored in the 65 to 74 range, earning the "Good" classification, and a very small proportion of 4%, or five students, scored between 75 and 100, placing them in the "Very Good" category. The results suggest that a minimal percentage of students are performing at a high level, while a substantial majority are underperforming, emphasizing the importance of implementing targeted instructional strategies to enhance students' comprehension and application of differential rules.

Table 3. *Correlation analysis of student performance and related variables*

Variables	Correlation coefficient	p-value	Interpretation
Test scores and prior calculus experience	0.45	<0.01	Moderate positive correlation, prior calculus experience is associated with higher scores
Test scores and confidence level	0.12	>0.05	Weak non-significant correlation; confidence does not strongly relate to scores
Test scores and study hours	0.52	<0.01	Moderate to strong positive correlation, more study hours related to better performance
Test score and attendance rate	0.30	<0.05	Moderate positive correlation, higher attendance is association with higher score

The correlation analysis in the table provides several key insights into the factors that affect student performance.

- There is a moderate positive correlation of 0.45 between test scores and prior calculus experience, with a p-value of less than 0.01. This indicates that students who have previous calculus experience

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tend to achieve higher test scores, and this relationship is statistically significant.

- The test scores and confidence level is quite weak at 0.12, with a p-value greater than 0.05. This means that confidence does not have a statistically significant impact on test performance, and therefore, confidence alone may not be a reliable predictor of student success in this context.
- A strong positive correlation of 0.52 between test scores and study hours, with a p-value less than 0.01. This suggests that students who spend more hours studying tend to perform better on tests.
- Attendance rate has a moderate positive correlation of 0.30 with test scores, with a p-value less than 0.05. This indicates that higher attendance is associated with higher test scores, emphasizing the importance of regular class attendance for better academic outcomes.

Student challenges in introduction to differential calculus

- Lack of Conceptual Understanding: Many students do not grasp what derivatives represent, which hampers their ability to apply rules correctly. Instruction should emphasize conceptual clarity alongside procedural skills.
- Difficulty in Applying Rules: Students often forget or misuse differentiation rules such as product, quotient, and chain rules. Reinforcement through practice and visual aids can enhance procedural fluency.
- Poor Mathematical Fluency: Frequent errors in calculations suggest limited practice or confidence. Regular, structured practice sessions are necessary to build fluency.
- Fear and Anxiety: Emotional barriers like fear of mistakes inhibit active participation. Creating a supportive classroom environment can encourage risk-taking and resilience.
- Lack of Practice and Repetition: Insufficient exposure to varied problems leads to weak skills. Incorporate diverse exercises and formative assessments.
- Misconceptions and Misinterpretations: Misunderstanding the purpose and interpretation of derivatives affects application. Use real-life examples and visualizations to clarify concepts.
- Need for Visual and Conceptual Aids: Visual tools such as graphs help in understanding the behavior of derivatives. Integrate graphing activities and technological tools into lessons.

5. CONCLUSION

From the analysis of the diagnostic test results and correlation data, it is evident that the majority of students at Milton Margai Technical University are struggling with the application of differential rules, with nearly 69% scoring in the "Very Poor" and "Poor" categories. Only a small fraction demonstrate strong proficiency, highlighting a significant need for targeted instructional interventions.

The correlation analysis further underscores the importance of prior calculus experience, study hours, and attendance in influencing student performance, suggesting that enhancing these areas could improve outcomes. Moreover the identified challenges; such as conceptual misunderstandings, procedural difficulties, lack of practice, and emotional barriers point to the necessity of adopting comprehensive teaching strategies. Incorporating visual aids, diverse practice exercises, and creating a supportive learning environment are crucial steps toward fostering better understanding and application of differential calculus among students.

These findings imply that curriculum developers and educators must prioritize active engagement methods, reinforcement of foundational concepts, and the integration of visual and practical tools to improve student mastery of differential rules. There is also a need to promote regular attendance and encourage study habits through motivating strategies.

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