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Three Point Perspective Techniques

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Abstract— There are many books that focus on one- and two- point perspectives, though only one geometric method is always employed. However, three point perspective is only demonstrated as a freehand illustration in all available resources, although there was a geometric method taught in some universities. Such method is complex as it implies non-orthoganal projections for the plan and elevation that makes it non practical. In other words, the elevation is projected at an angle which shows two sides of the object, and the plan is also projected of a tilted object that shows all sides. And they have to be produced prior to constructing the perspective. This is why most artists would rather use freehand estimation to plot three point perspective.

In this research, I will demonstrate many different methods. Some use the (station point), others don't. Some don't rely on vanishing points if they become too far to locate within the drawing board. There is also a mechanical method that plots the perspective within a pre-defined square using horizontal and vertical coordinates only without using any other elements, such as, vanishing points, horizon line, etc. All methods employ conventional (Plan View) with or without a conventional elevation. The methods are simple and don't involve angular elevations of two sides at all.

Keywords— geometric, three point, freehand illustration, mechanical method, horizontal coordinate, vertical coordinate.

INTRODUCTION

Three point perspective or so-called Bird's eye- and Worm's eye view happens when the camera is tilted downwards or upwards as shown in Fig. 01. The tilt angle of the camera (denoted by nº) has never been used in any perspective procedure available. The value of this angle (nº) determines the locations of the vanishing points and establishes different perspective views. There are three concrete basics to rely on, in order to understand the procedures of three point perspectives.

1. The definition of Picture Plane i.e. Projection Plane.
2. The definition of Vanishing Points and how they are located.
3. The basic Perspective Theorem established by me in 1979.

Upon understanding these definitions, the entire research will be clear.

Fig. 01. *Bird's eye view & Worm's eye view.*

1. Projection Plane (Picture Plane)

Projection plane is in some likeness of a negative film inside a camera on which an image appears. Projection plane can also be described as a window pane from which a person standing at arm's length traces upon the glass the scene outside. A person doing this will be drawing a picture in perspective (i.e. a 3D projection), as shown in Fig. 02.

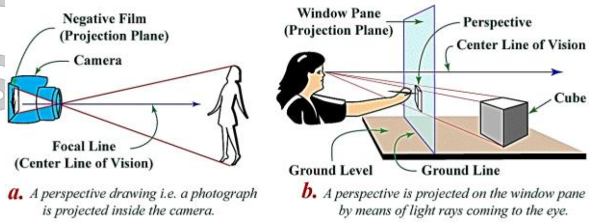

Fig. 02. *Basic Interpretation of the Perspective Projection and Projection Plane.*

If the camera is tilted, then, its negative film (i.e. Projection Plane) will be tilted too. Thus, the window pane will be tilted as well, as it represents the negative film as a projection plane. Fig. 03.

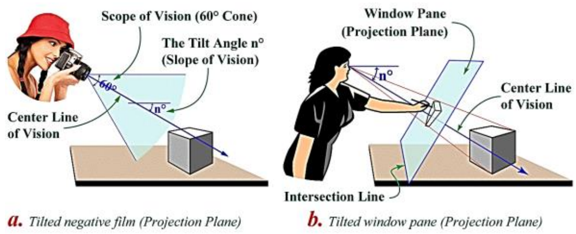

Fig. 03. *If the camera is tilted, Projection Plane must be tilted.*

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2. Vanishing Points (VP)

Vanishing points are simply expressed as the points on picture plane that correspond to the farthest points of directed straight lines which are receding from the observer to infinity. To clarify such definition, Fig. 04 shows the vanishing point of the tightrope (ad). The observer who is looking at points (a, b, c and d) through the window pane (i.e. Picture Plane) will see the projections of these points on the window pane as (A, B, C and D) respectively. If point (d) goes farther and farther on the same path of (ad) until it reaches infinity, its light ray coming to the eye will tend to parallel the tightrope. It is mathematically considered that the parallel lines tend to converge in infinity. When the infinite light ray intersects picture plane, it will determine the location of the vanishing point of the assumed tightrope.

What happens when the ground level is tilted? In Fig. 05, (ab) placed on Ground Plane 1 at any direction intersects Picture Plane at point (a). As rotating Ground Plane 1 about Ground Line, point (b) will sweep a plane (dbB) perpendicular on both Picture Plane and Ground Plane 1. Thus, the vertical projection of (aB) on Ground Plane 1 is (ac). The Horizon Line of Ground Plane 2 is determined by intersecting a plane parallel to Ground Plane 2 and passing in the eye (e) as with Picture Plane. Finally, line (sv) is drawn parallel to (ac) from Station Point (s) to Picture Plane. As (v) is projected up to the Horizon Line of Ground Plane 2, it will locate the vanishing point of line (aB).

3. Perspective Theorem

Any straight line intersecting the picture plane has one corresponding straight line in perspective view, drawn between the vanishing point of the assumed line and its intersection point on picture plane.

If a straight line lying on a ground plane in any direction (Fig. 06) intersects the picture plane, the intersection point will represent itself in the perspective view, since it is one of the points of picture plane. Then, a vanishing point for that line can be located on picture plane that corresponds to the other end of the straight line tending to infinity.

Therefore, the line segment on picture plane between the intersection point and VP corresponds to the assumed line, as starting from the intersection point and receding to infinity. Furthermore, any point located on the assumed line has an equivalent point in the perspective drawing located on the line segment between the intersection point and the vanishing point.

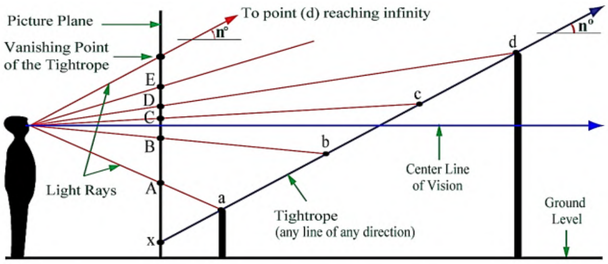

***Fig. 04.** Side view clarifying the vanishing point (VP) of the tightrope (ad).*

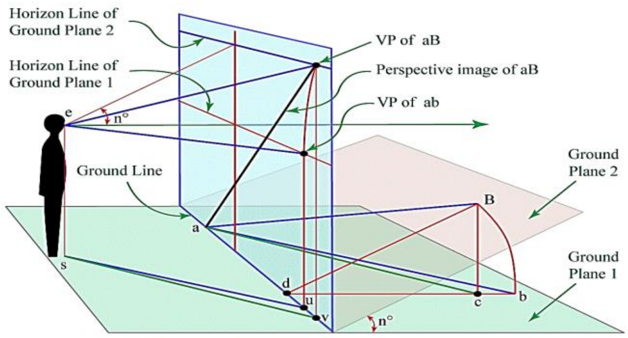

***Fig. 05.** An illustration showing a vanishing point of a line placed on a tilted Ground Plane 2.*

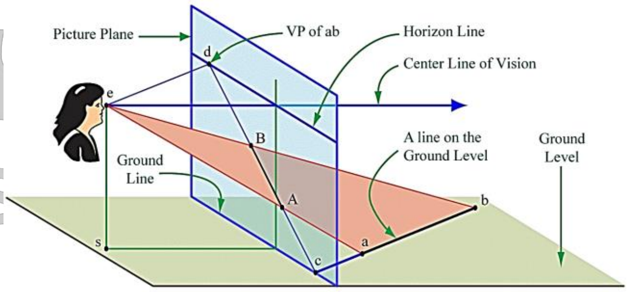

***Fig. 06.** Isometric drawing clarifying the perspective theorem.*

In Fig. 06, line (ab) lies on the horizontal ground level and meets with picture plane by point (c). Line (ab) also has the vanishing point (d) which is defined by drawing line (ed) from the eye point (e) parallel to the assumed line (ab). If a plane rests on the parallel lines (cb and ed) it will intersect the picture plane making the intersection line (cd). Thus, line (cd) is the perspective image of line (cb) receding from point (c) to infinity on the ground plane. Axiomatically, points c, a and b have their corresponding points c, A and B respectively, in perspective view, i.e. on line (cd) on picture plane.

4. Example

Fig. 07 is based on Fig. 05 to define the locations of the vanishing points, where a square is placed on the

United International Journal for Research & Technology

Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832

horizontal Ground Plane. Since any point on the axis of rotation is the same whether ground plane is horizontal or not, the perspective theorem can be equally applied. The vanishing points are simply defined by setting the line (st) at 40° (considering the camera tilt angle is 40°). Then the left and right VPs are projected down on the red horizontal line from (t). The eye level is on the Horizon Line of level plane. The distance between both horizon lines is defined by setting the green line at 40° angle. The angle (40°) is placed at the same distance as the station point (s) from the Horizon Line of level plane (namely, pq = sq). Try to compare Fig. 07 to Fig. 05. Once the vanishing points are located on the Horizon Line of the tilted plane, the perspective drawing of the tilted square is drawn by applying the perspective theorem in Fig. 06.

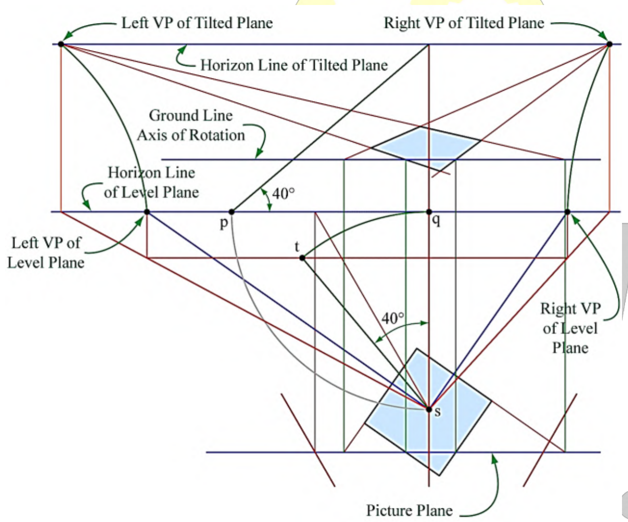

Fig. 07. *A square on a tilted plane is drawn by using its vanishing points, and applying the perspective theorem.*

5. Defining the Verticals

In Fig. 08, the perpendicular line (bc) is rotated about Ground Line defining line (BC) vertical to Ground Plane 2. Line (BC) has a vanishing point defined by a parallel line drawn from (e) to Picture Plane. This vanishing point will definitely be located on the vertical center line, because (bc) sweeps a plane perpendicular on Picture Plane when rotated about Ground Line. As (bc) is the height of a perpendicular rectangle (abcd) that intersects Picture Plane to form (ad), then, rotating the rectangle (abcd) around Ground Line will make the rectangle (aBCD) perpendicular on Ground Plane 2. Observe that rotating (d) about the Ground Line makes the triangle (adD) perpendicular upon Picture Plane and projected towards the observer. As point (a) represents itself in perspective, thus, line (aD1) will be the perspective image of line (aD). If a line is drawn between (D1) and (VP of line DC) it will represent (DC) in perspective, and one point on that vanishing line will represent (C) in perspective as well.

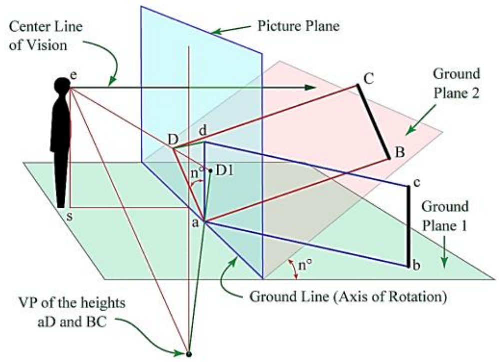

Fig. 08. *An illustration showing how the verticals are defined.*

Fig. 09 is based on Fig. 08, where the upright triangle (aDk) is placed on the axis of rotation. Observe that Dk has VP (E). Dd has VP (G), and Dg has VP (H). (ef) is the top view of a vertical plane, and when rotated 40°, its vanishing point becomes (F). (D1) is defined in three methods. You can easily see that (D1) is the intersection point of four lines (K D1, H D1, G D1 and E D1). (e1 D1) is the perspective image of the vertical above (e). If the elevation is set tilted, you either use (Dk to e4) to define its height relying on VP (E), or use (Dg to e2) relying on VP (H). Now, if the elevation is upright (as aD), then extending to (e3) will define (D1) through VP (G) which is always set at half the tilt angle (n°) namely 20°. Thus, the plane (e1-D1-F) is the perspective representation of the upright plane (ef) receding from point (e) to infinity.

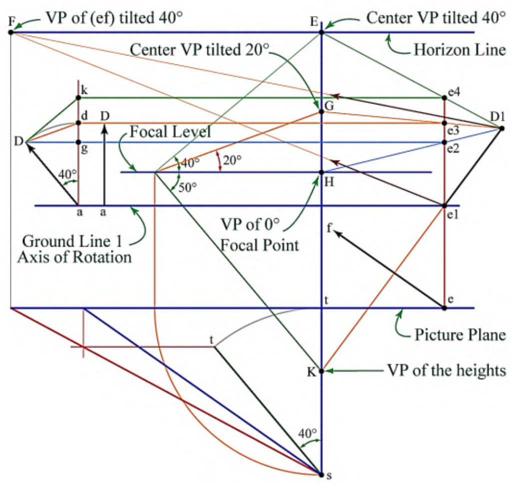

Fig. 09. *The perpendicular plane (ef) of height (aD) is defined in perspective, using three methods.*

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6. Basic Procedure of Three Point Perspective

In Fig. 10, the vanishing points are first located directly on the horizon line, considering the tilt angle (n = 40°). The triangle (aDk) is identical to (aDk) in Fig. 09, where (aD) is the height of the cube set at 40° from the vertical (ak). Using the perspective theorem, lines are extended from the plan to Picture Plane and the intersection points are projected up to Ground Line 1 (Axis of Rotation). (e1) is used to define the height of the cube. The line (e1D1) is extended from VP of the heights to represent (aD) in perspective as established from (e1). Once point (D1) is located, the rear side of the cube is defined first. With the help of the left and right vanishing points the whole perspective view of the cube is done.

This technique is such that defining one height as (e1D1) and the bottom of the cube drawn from the intersection points on axis of rotation. What if another level, such as, the top of the cube is required to draw first from its intersection line (i.e. Ground Line 2), would it be possible?

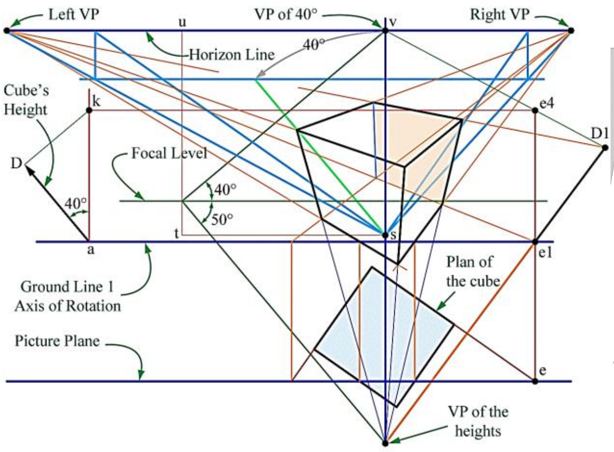

Fig. 10. *A basic perspective procedure of a cube as a Bird's Eye View.*

7. Another Procedure Using Multiple Ground Planes.

Ground planes are considered all the planes parallel to the Ground Level. They can be used to define different heights in perspective. Fig. 11, below, is a side view showing an object (abcd) rotated around the axis (e) defining its new position as (ABCD). If we consider (g) as the axis of rotation for the top (DC), then, rotating it back to its plan will make the plan offset from the plan of (ab). In order to make both plans (namely top views) applied on each other, we have to push both the observer and Picture Plane (as both represent a camera) forward at a distance (gi) equal to (fg). If (i) is the axis of rotation for (D1C1), then its plan will be right on top of (ab) when rotating back to its upright position. To make it easy to imagine this situation, just look at both observers (the blue and red) with respect to their Picture Planes (blue and red). The blue observer will see (DC) on Picture Plane 1 the same as the red observer seeing (D1C1) on Picture Plane 2. Practically, we are drawing the twisted cube (ABC1D1) as being rotated about two axes. But, we compensated the deformity by moving the camera (or the observer) forward at a distance equal to the opposite of the tilt angle (n°), where the adjacent is the true height, and the hypotenuse is the distance between the Ground Lines.

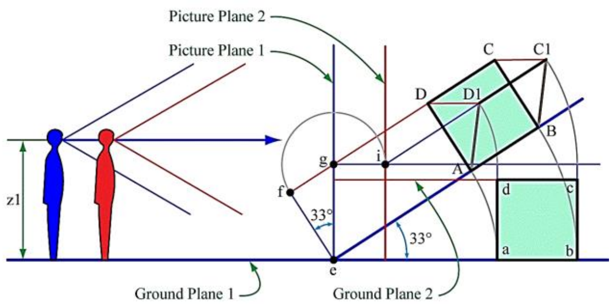

Fig. 11. *A side view showing a technique to deal with multiple levels in three-point perspective.*

The purpose of this method is to avoid the unconventional - hard to do – plan projection of a tilted object, as illustrated in the following Fig. 12.

A complex process is employed to define the Top Projection of a tilted cube (at 33º) as shown in the upper illustration of Fig. 12. As the axis of rotation must be set horizontal in perspective, the Top Projection is rotated and positioned in the bottom part. The same triangle (efg) defined in Fig. 11 is placed on Axis of Rotation, so that the distance between both Picture Planes (fg) is defined.

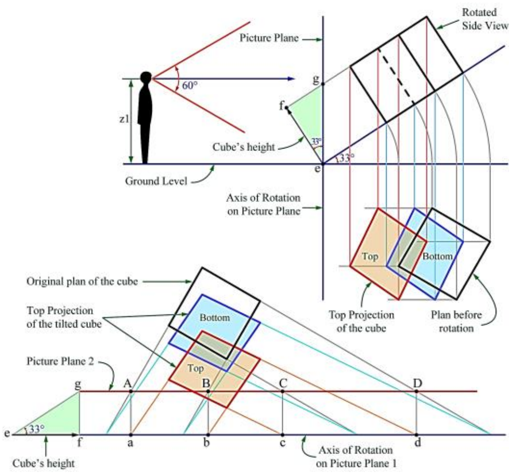

Fig. 12. *Setting two Picture Planes to avoid Unconventional top projection of a tilted object.*

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Observe that the intersection points (a, b, c, d) of the pink (Top) lines are aligned with the points (A, B, C, D) on the red Picture Plane 2 which are also the intersection points of the lines extended from the original plan of the cube.

The idea herein is to avoid using such complicated and unconventional top and side projections. Imagine how hard is to draw a top projection (or a plan) of a tilted house prior to setting up the perspective procedure. With this method, you don't need the top projection of a tilted object. The lines extended from the original plan will define the top of the cube when intersecting Picture Plane 2, and will too locate the bottom of the cube when intersecting the Axis of Rotation.

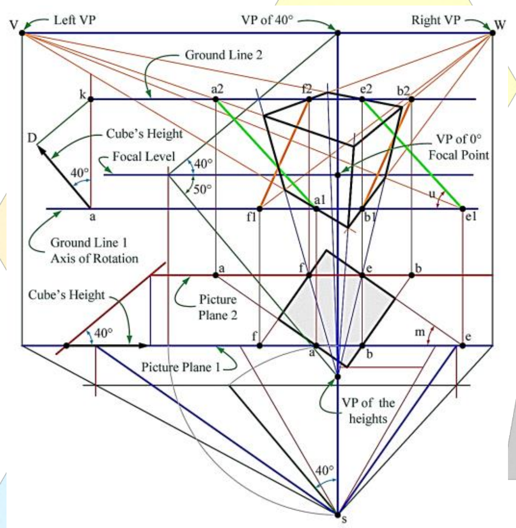


Fig. 13. *Using two picture planes to define different levels in Three Point Perspective.*

8. Cube defined by two Ground Planes

In Fig. 13, Picture Plane 2 is defined by the right-angled triangle resting on Picture Plane 1, where the adjacent is the cube's height and the opposite is the distance sought between both Picture Planes. The camera tilt angle (n°) is 40°. Using the perspective theorem, lines are extended from the plan to both Picture Planes and then projected up to the Ground Lines. Of course, Ground Line 2 is the intersection line of Ground Plane 2 with Picture Plane which is the horizontal line established from point (k). The perspective view of the cube is simply done by connecting lines from the intersection points on both Ground Lines to their vanishing points.

9. Another Example Using Intersections of the Cube's Sides

You may notice in Fig. 13 that VP of the heights was not used in the process. That method will be ideal when the tilt angle (n°) is small enough to make (VP of the heights) too far to locate on the drawing board. The heights are simply drawn connecting the top and bottom of the cube. Observe the thick lime green lines (a1a2, e1e2) and the thick orange lines (b1b2, f1f2). They look parallel at certain angle. What are they? In fact, they are the intersection lines of the sides of the tilted cube as with Picture Plane. This may give us another procedure to draw a perspective without relying on multiple Picture Planes. The only problem is defining the angle of these intersection lines, such as, e1e2 and f1f2. There are no other methods to define those angles except the method shown in Fig. 13 itself. However, I would rather use this simple equation,

tan u = tan m / sin n

(Proven in Fig. 13, using simple Trigonometry)

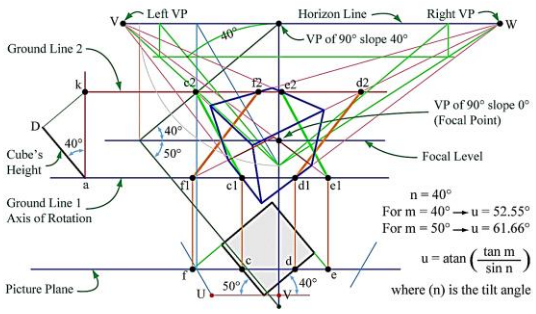


Fig. 14. *Three point perspective of a cube established by using the intersection lines of the cube's sides as with Picture Plane.*

In Fig. 14, the tilt angle (n°) is equal to 40°. The plan of the cube is set at 40° and 50° angles. Applying the equation on a calculator, we get the values of (u) shown in Fig. 14. A second Picture Plane was not necessary and VP of the heights was not required either. The cube is simply drawn by using the intersection points f, c, d and e sent up to the axis of rotation defining the points f1, c1, d1 and e1.

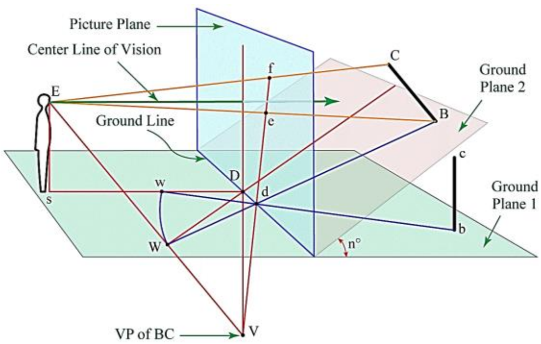


Fig. 15. *An illustration clarifying a method, using station point (W).*

United International Journal for Research & Technology
Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832

Those points define the bottom of the cube.

For the top of the cube, the orange and lime thick lines are drawn between Ground Lines at 61.66° and 52.55°. Then, the cube's top is drawn by the vanishing lines extended from the intersection points c2, f2, e2 and d2. It is a very simple technique of three point perspective.

10. Using the Station Point in Three Point Perspective

See Fig. 15 and Fig. 16. There are two case scenarios to consider. The Station Point is the vertical projection of the eye (E) on either Ground Plane 2, as in Fig. 15, or on Ground Plane 1, as in Fig. 16. In Fig. 15, the Station Point on Ground Plane 2 is point (W). Now, (EW) is parallel to the height (BC). Therefore, the plane (EVBC) rotating about (EV) will intersect Picture Plane defining radii, such as (Vf) established from VP of the heights (V). Observe that line (df) has two line segments aligned on it, (de) corresponding to (dB) and (ef) corresponding to (BC). Thus, the depth points as well as heights will all be located on the radius Vf.

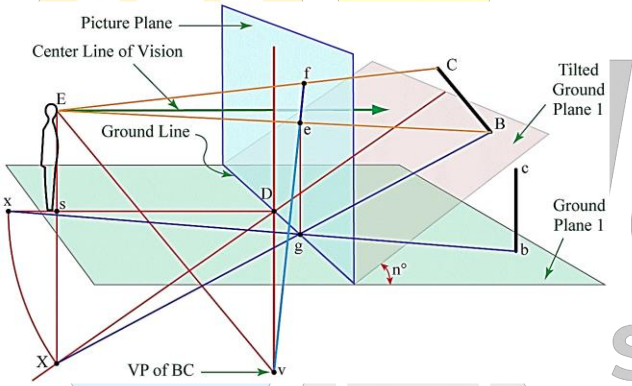


Fig. 16. *An illustration clarifying a method, using station point (X).*

In Fig. 16, station point (X) is involved. When the perpendicular plane (EXB) rotates about (EX) it will intersect Picture Plane defining vertical lines, such as (ge) on Picture Plane. Therefore, (ge) represents (gB) in perspective. Observe that taking another tilted Ground Plane at (C) will make (X) at shorter distance to (BC). The difference will be equal to the distance between two Picture Planes if it is set for Ground Plane 2. So keep that in mind when applying this method. On the other hand, re-locating Picture Plane in Fig. 15 will not make any difference.

11. Example

In Fig. 17, the cube is drawn using both techniques clarified in Fig. 15 and Fig. 16. The vertices of the cube, such as, (A and b) can all be located on the verticals established from picture planes. For instance, if (b) is extended to Picture Plane 1 by using (x), it will locate corner (b) on the vertical (gb). And if (a) is extended to Picture Plane 2 by using (y), it will locate corner (A) on the vertical (hA). Observe that both procedures yield the same result. For example, if we use the point (w), which is always measured from the axis of rotation on Picture Plane 1 (based on the method of Fig. 15), then (we) from (b) will locate (e2) which defines the cube's edge at (v e2). And (wd) from (a) will locate (d1) which defines the cube's edge at (vA).

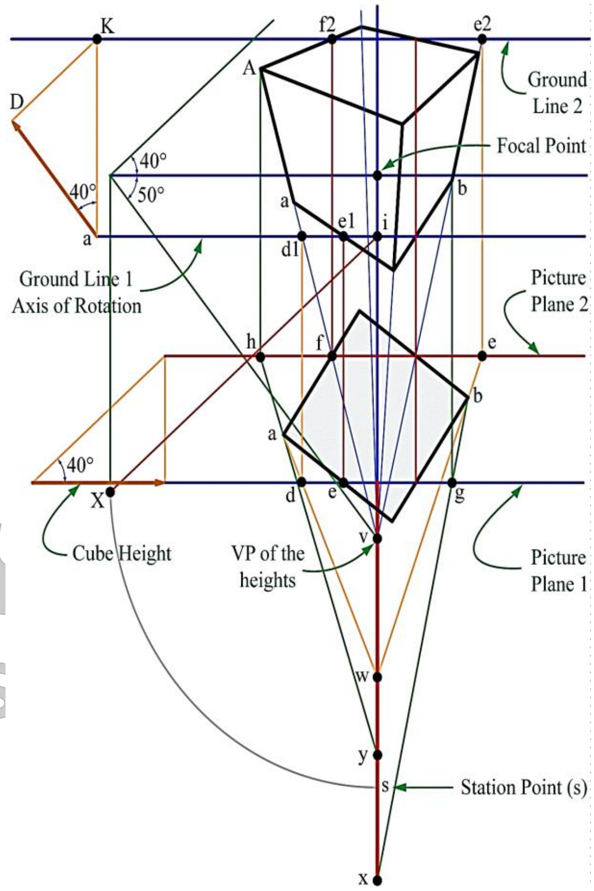
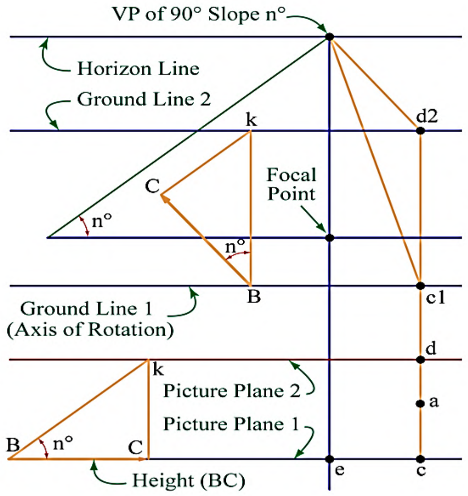


Fig. 17. *Applying the techniques*

12. Dealing with Vertical Lines in the Plan

The vertical lines in the plan are the simplest to define in perspective, as they represent the vanishing lines receding to the Center VP on the Horizon Line, regardless of the tilt angle of the camera (nº). Why? By relying on the equation (tan u = tan m / sin n), line (cd) is set vertical in Fig. 19 (where m = 90°). Thus, (c1 d2) will be vertical according to the equation. For more clarification, look at the 3D illustration in Fig. 18. The intersection line (bd) remains vertical as (be) after rotating the vertical plane (abdc) about the Ground Line if and only if (ab) is vertical to the Ground Line.

United International Journal for Research & Technology
Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832



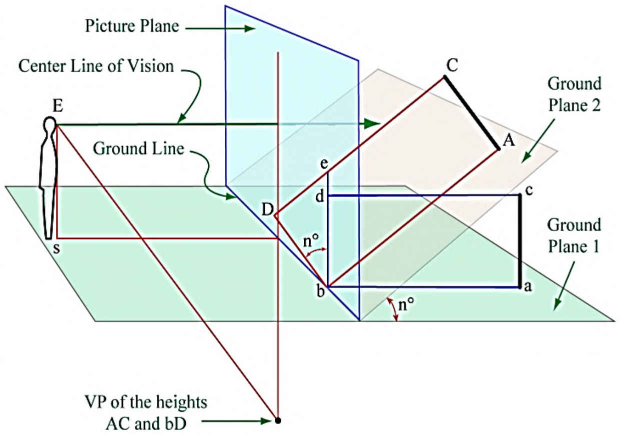


Fig. 19 *Fig. 18*

Fig. 19. *Basic procedure is used to draw the vertical lines in the plan.* ***Fig. 18.*** *An illustration showing that the intersection line (be) remains vertical if and only if (ab) is vertical to the Ground Line.*

13. Bird's Eye View of Two Point Perspective

This example in Fig. 20 involves horizontal and vertical lines of the plan. Firstly, (tw) is defined equal to (xy). Then, a line, such as, (wd) is drawn at any direction whose perspective image is (vz) established from the intersection point (e), as based on Fig. 15. Then, horizontal lines are drawn from the plan defining (a, c and d). The procedure of Fig. 19 is then applied to these points to define ridges and points in the perspective drawing. This procedure is quite clear and simple.

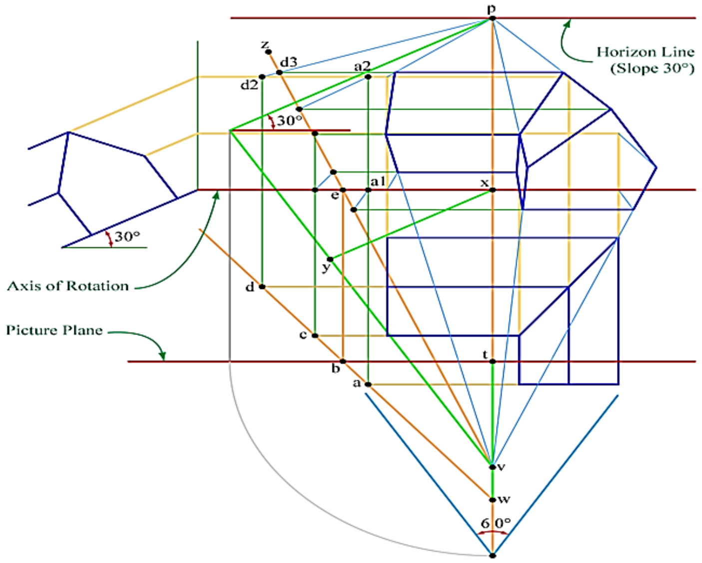
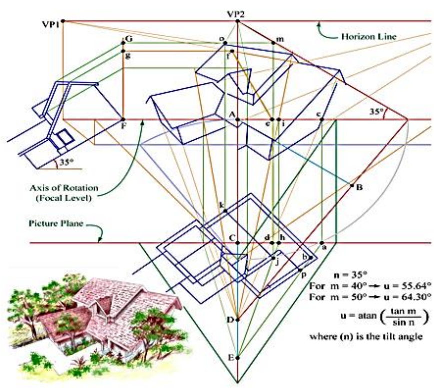


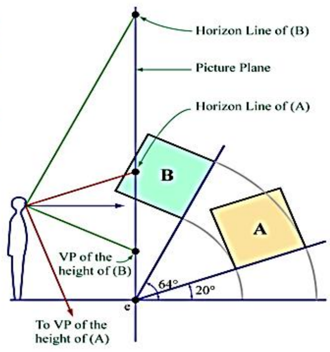
Fig. 20. *An outline of a house drawn by using the procedures in Fig. 15 and Fig. 19.*

14. Bird's Eye View of Three Point Perspective

This example, shown in Fig. 21, implies the procedures used in Figures 14, 17 and 19. I emphasized on the roof part as how the lines are directly defined, using different techniques. For instance, the roof ridge (jk) is drawn in perspective by its end points (j) and (k). Using the methods in Fig. 17 and Fig. 19 on (j), where (j) is extended to (h) then to (i) to define the line (Di) extended from (D) to the upper roof ridge. Meanwhile, (j) is extended vertically to (m) at height (G) and then through (VP2) to (Di) based on Fig. 19. Then, the vanishing line from (VP1) is drawn. Point (k) is vertically extended to (o) and deflected from (VP2) intersecting the vanishing line from (VP1). However, the roof ridge from (p) is defined using the method of Fig. 14, as (p) is traced to (d), then (e) and (f). (ef) is set at 64.30° since (pd) has 50° angle.

United International Journal for Research & Technology
Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832





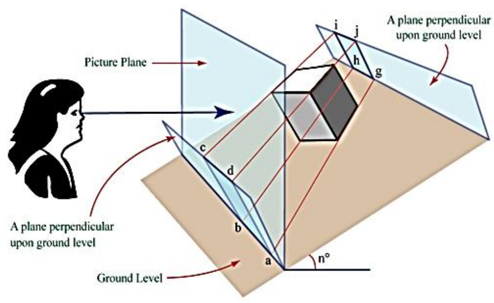


Fig. 21 *Fig. 22* *Fig. 23*

***Fig. 21.** A three-point perspective of a house Plotted directly through various techniques**. Fig. 22.** An illustration clarifying a range of the tilt angle (n°) smaller than 30° or greater than 60°. **Fig. 23**. An illustration clarifying a procedure that involves front and rear projections as an alternative to locating the Horizon Line in Three Point Perspective.*

15. Vanishing Points outside the drawing board

We have seen many examples in which the tilt angle (n°) was set in vicinity to 45°, somewhere between 30° and 60° angles. Such setting is the simplest to draw, since all the vanishing points including VP of the heights can be located on the drawing board. However, when the tilt angle (n°) is arbitrarily set smaller than 30° or bigger than 60° (as shown in Fig. 22), consequently, either VP of the heights or the Horizon Line will become too far to locate on the drawing board. We can easily exclude VP of the heights by using a second Picture Plane, or by implying the intersection lines of the Object's sides using the equation (tan u = tan m / sin n). Upon excluding the Horizon Line, all vanishing points will be ignored. The best procedure to use when (n°) is greater than 60° is to place the object within a box positioned in such a way as in Fig. 23, and using projections applied on the front and rear sides of the box.

16. A Cube Drawn without Defining the Horizon Line

Fig. 24 is simply constructed, based on Fig. 15 and Fig. 19, using VP of the heights and the Focal Point as follows:

1. The box (ihgf) is set where the front side must be at the axis of rotation on Picture Plane.
2. The important point to emphasize herein, is the shape (AabB). It is the side view of the box rotated 40° as it is the camera tilt angle (n°). The distance (ab) is equal to (gh). Both (aA) and (bB) are equal to the height of the cube.
3. The link between the front and rear sides is through (gh) which is extended to (j). The horizontal lines extended from (a, A, b and B) to (hj) are deflected to the Focal Point intercepting the red heights. The method of Fig. 15 is applied to define the far corner (h), as well as, the front and rear projections (shaded in grey) which are (f1f2 d2d1) and (e1e2 c2c1). Then, the front and rear projections are connected by lines which are intercepted by the heights to form the cube.
4. This is the simplest procedure to draw any object in three-point perspective without relying on the vanishing points.

17. A worm's eye view of a chair

Fig. 25, shows a worm's eye view of a chair. This is quite a simple example. All the curved lines are drawn restricted within rectangles, as shown in the side view of the chair. I only defined all the rectangles in the perspective structure by using the basic method of Fig. 10. Then, I filled in the curves as confined within those rectangles in perspective. This chair has three vanishing points on the Horizon Line, because two sides (cf) and (de) of the chair are not parallel, as shown in the plan view. The technique is similar to Fig. 10, except that the elevation (i.e. the side view) is set upright. Thus, the horizontal lines extended from the elevation to (ab) must be deflected to vanishing point (A) which is set at half of the tilt angle (17.5°). As the points are defined on the corresponding height (bV), the entire perspective drawing is done with the help of the three vanishing points on the Horizon Line. Then, freehand touches are put into it to fill in the curves.

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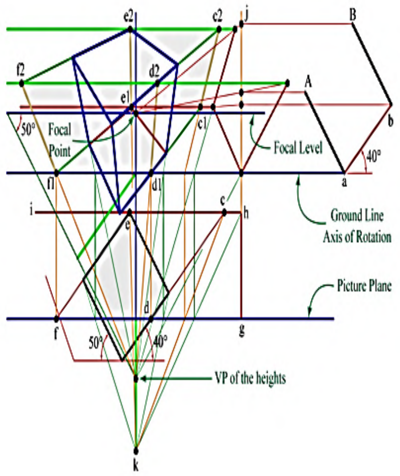

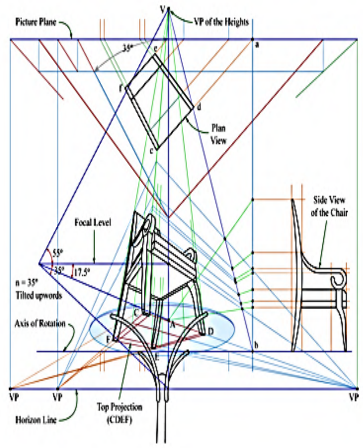

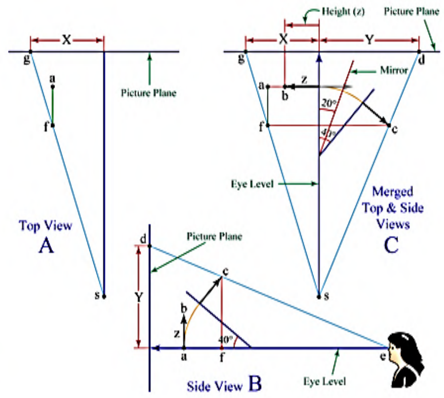

Fig. 24 *Fig. 25* *Fig. 26*

Fig. 24. *A cube drawn without relying on the upper vanishing points as the Horizon Line is undefined.* ***Fig. 25.*** *A worm's eye view of a chair done by using the method of Fig. 10.* ***Fig. 26.*** *The mechanical method is built by merging the top and side views of a rotated point (c) in space.*

18. A Mechanical Method

This method was applied in my machine patented in England in 1981. In Fig. 26-B, Axis of Rotation is best located at the center of the object at the Eye Level. The height (z) above or below the Eye Level will set the location of the Ground Level desired. In illustration (B), point (b) is set at height (z) over point (a) in the plan. It is then rotated at (n = 40°) defining (c). As it is projected to Picture Plane it defines (d) which is the perspective image of (c). In the top view (A), point (a) is defined at the same distance from (s) as it is from (e) in the side view (B). Illustration (C) combines (A) on the left to (B) on the right. Now, (f) is the top projection of (c) on the plan, which is projected to (g) defining the X-coordinate. You may notice that I reversed the direction of (z) so that (b) became mirrored to (c) as the mirror is set at 1/2 n°.

19. Applications to the Mechanical Method

Fig. 27, shows that one of the vertices of the cube is defined. All other vertices are equally defined. You can notice that the perspective view is filling all the square and vanishing points have never been determined. The procedure is quite simple.

You can follow it from point (a) in the plan to (b, c, d, e) for the horizontal coordinate of (A), and from (a) to (b, c, f, g) to define the vertical coordinate of (A). Keep in mind that (de) is always set at 45° angle. The most important part of this procedure is setting the red line (xy) at half of the tilt angle of the camera (n°). Notice that both center lines of the square are practically considered as the eye level. Thus, the height of the cube is set horizontally with respect to the center line (sx). Thus, point (b) defines the base of the cube at (a). Then, point (b) is mirrored to (c), where the mirror (xy) is set at half of (n°) i.e. 20°. That is the whole procedure.

If the mirror (xy) is set on the opposite side of the vertical line (sx), then, the resulting perspective view will be a worm's - eye view, as shown in Fig. 28.

20. Mechanical Method when Tilt Angle (0°)

In Fig. 29, the same procedure is applied, as mirroring through the vertical position. You may notice that mirroring doesn't make a difference except shifting from right side to left side. Thus, you don't have to use the mirror at all. However, for a machine, the process has to involve a rotating mirror in order to be able to generate all kinds of perspective views, for all tilt angles (n°) sought.

21. Zooming in / out

This method also allows you to set the size of the resulting perspective drawing by just changing the size of the square and positioning it relative to the station point, as shown in Fig. 30.

In Fig. 30, notice that Picture Plane (i.e. the upper edge of the square) is located closer to the Station Point (s). Thus, the resulting perspective drawing has become smaller. The farther Picture Plane from the Station Point (s) is, the bigger the perspective view is.

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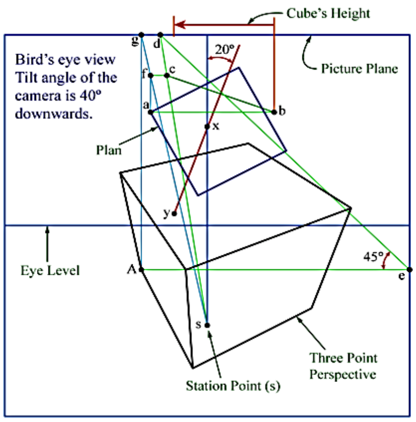
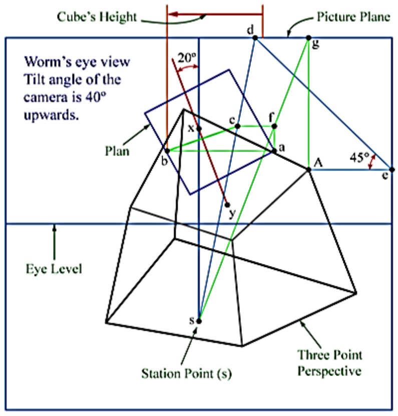
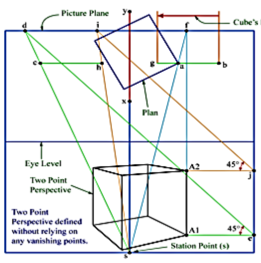
Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832







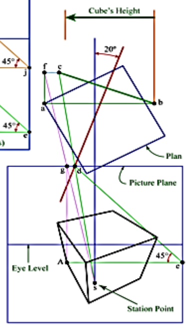


Fig. 27 *Fig. 28* *Fig.29* *Fig.30*

Fig. 27. *A bird's eye view done by using horizontal and vertical coordinates.* ***Fig. 28.*** *A worm's eye view done by using horizontal and vertical coordinates.* ***Fig. 29.*** *Two point perspective view done by using horizontal and vertical coordinates.* ***Fig. 30.*** *The size of the final > perspective drawing depends on the square size with respect to the station point (s).*

22. Mathematical Approach

Notice that the mechanical method is related to the Cartesian coordinate system. Thus, it is quite possible to establish mathematical equations that define the (X,Y) coordinates of any point in perspective. Although the diagram in illustration (C) of Fig. 26 can be used to define such equations, I found it easier to rely on the method demonstrated in Fig. 16, as shown in Fig. 31.

In Fig. 31, observe that the origin of (x,y,z) coordinates of the object is at the camera lens- or the observer's eye; whereas the origin of the perspective coordinates (X,Y) is at the focal point on

the projection plane (namely Picture Plane). Through experimenting of many alternative locations of the Origin points, I determined that the simplest equations obtained are in this setting in which the camera lens and the focal point are used as the origin points. Taking the top and side views of Fig. 31 yields Fig. 32, which is used to prove the equations with simple trigonometry.

In Plane Geometry (See Fig. 32):

$$\frac{bk}{z} = \tan n \qquad \text{Thus,} \qquad bk = z\tan n$$

$$\frac{z}{ak} = \cos n \qquad \text{Thus,} \qquad ak = \frac{z}{\cos n}$$

$$\frac{F}{ec} = \cos n \qquad \text{Thus,} \qquad ec = \frac{F}{\cos n}$$

$$\frac{gc}{F} = \tan n \qquad \text{Thus,} \qquad gc = F\tan n$$

$$\frac{dc}{ak} = \frac{ec}{ek} \qquad \text{Thus,} \qquad dc = \frac{(ak)(ec)}{ek}$$

$$Y = gc - dc \qquad \text{Thus} \qquad Y = gc - \frac{(ak)(ec)}{ek}$$

Substituting in,

$$Y = F\tan n - \frac{\left(\frac{z}{\cos n}\right)\left(\frac{F}{\cos n}\right)}{y + z\tan n}$$

$$Y = \frac{F\tan n(y + z\tan n) - \frac{Fz}{\cos^2 n}}{y + z\tan n}$$

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United International Journal for Research & Technology
Volume 04, Issue 06, 2023 | Open Access | ISSN: 2582-6832

$$Y = \frac{Fy \sin n \cos n + Fz \sin^2 n - Fz}{y \cos^2 n + z \sin n \cos n}$$

Observe that (z) and (n) are negative, thus,

$\sin n = -\sin(-n)$ and $\cos n = \cos(-n)$

$$Y = \frac{-Fy \sin n \cos n - Fz \sin^2 n + Fz}{y \cos^2 n + z \sin n \cos n}$$

$$Y = \frac{Fz(1 - \sin^2 n) - Fy \sin n \cos n}{\cos n\,(y \cos n + z \sin n)}$$

$$Y = \frac{Fz \cos^2 n - Fy \sin n \cos n}{\cos n\,(y \cos n + z \sin n)} \quad \text{Thus} \quad Y = \frac{F(z \cos n - y \sin n)}{z \sin n + y \cos n}$$

In the Top View,

$$\frac{X}{x} = \frac{fh}{ah} \quad \text{Thus,} \quad X = \frac{x(fh)}{ah}$$

$$X = \frac{\frac{xF}{\cos n}}{y + z \tan n} \quad \text{Thus,} \quad X = \frac{Fx}{z \sin n + y \cos n}$$

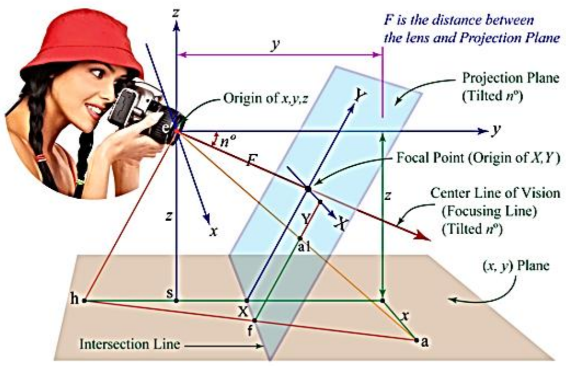

Fig. 31

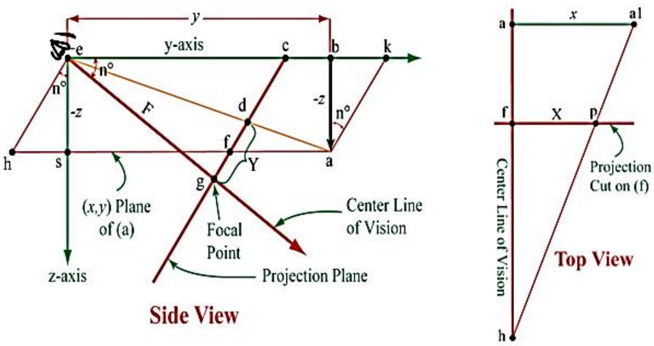

Fig. 32

Fig. 31. *The perspective procedure used to locate point (a1) on the Projection Plane.* ***Fig. 32.*** *Top and Side Views showing the representation of (a) on the Projection Plane as (d).*

Where:

(x) is the x-coordinate of a point (a) in the plan from the Center Line of Vision.

(y) is the y-coordinate of a point (a) in the plan from the Station Point (s).

(z) is the z-coordinate of a point (a) in the elevation from the Eye Level.

(X) is the X-coordinate of a corresponding point in perspective from the Center Line of Vision (or Focal Point).

(Y) is the Y-coordinate of a corresponding point in perspective from the Focal Level (g).

(F) is the Focal Length (a magnification factor).

(n°) is the tilt angle of the camera.

24. Example 1 (n = 0°)

In Fig. 33, the plan of the cube (abcd) is positioned with respect to the station point which is the origin (0) of (x,y) coordinates, and the x and y coordinates are recorded in the table, below. There are two altitudes, (z1 = -0.93”) and (z2 = -0.32”) which are involved in the equation (Y = Fz/y); whereas the other equation does not have z-coordinate. You can assume any value for (F). (F) defines the position of the Projection Plane. The greater (F) is, the bigger the resulting perspective is. Plotting X– and Y-coordinates of eight points taken from the table, the perspective drawing of the cube is defined by connecting all the vertices. When n° = 0°, (X = Fx / y) and (Y = Fz / y), since (sin 0 = 0) and (cos 0 = 1). The perspective coordinates (X and Y) are generated automatically by using Microsoft Excel file which is programmed to execute the equations.

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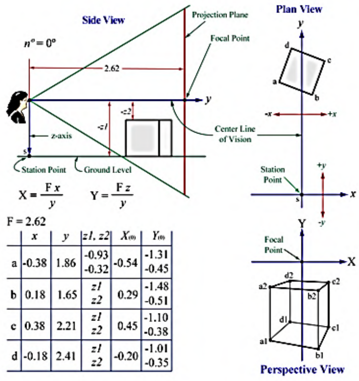


Fig. 33

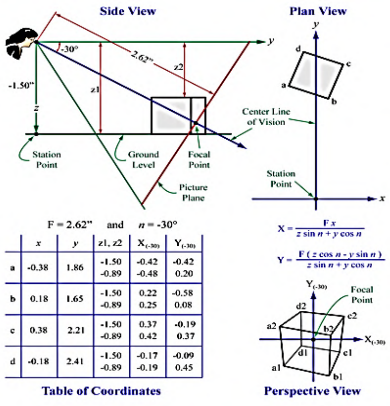


Fig. 34

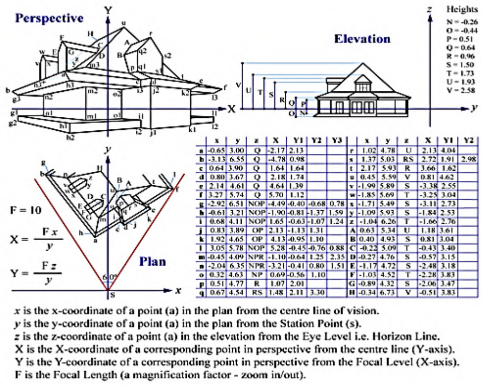


Fig.35

Fig. 33. *A cube drawn using the equations to locate the points.* ***Fig. 34.*** *Using the equations to draw a bird's eye view of a cube.* ***Fig. 35.*** *A house is drawn using the equations to locate the points.*

You can download my Excel file here: https://drive.google.com/file/d/14fczx3h4TosEAYVCuJ9reCpEIqPGb_9y/view?usp=drivesdk

25. Example 2 (n = -30°)

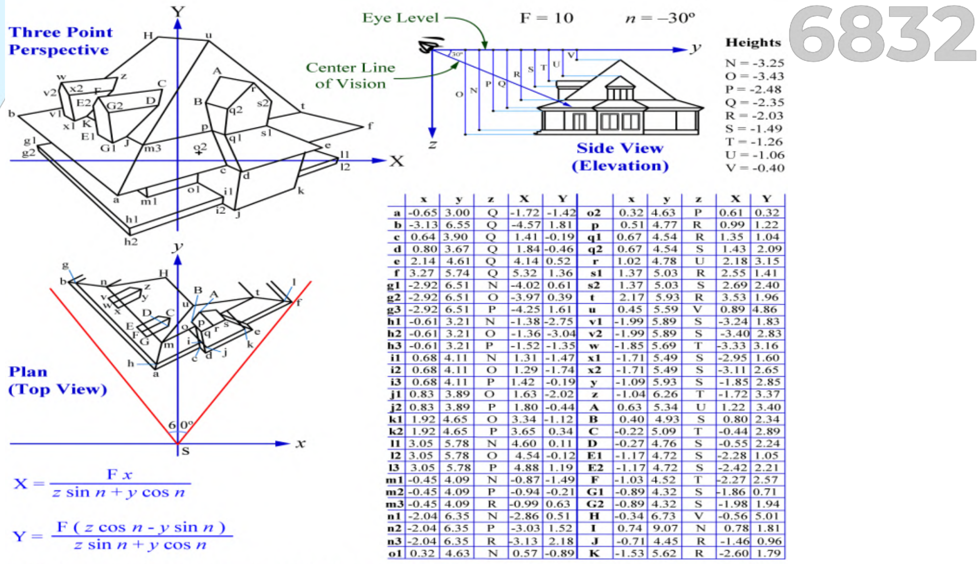
The cube's coordinates (x, y, z) were entered in Microsoft Excel file in order to output perspective coordinates (X and Y), as shown in Fig. 34.

26. Applying the Equations to Draw a House (n = 0°)

Observe that any point in space can be defined by these equations, whether it is a part of a straight line or a curved line. The perspective view is plotted by the points of X and Y coordinates calculated by the equations, where x and y coordinates are taken from the plan, whereas z-coordinates are taken from the elevation, as shown in Fig. 35, below. If you are a professional drafts-person who does artistic renderings of architectural projects you will definitely appreciate these equations, as they will be a truly time saving procedure. You do not have to plot every point in perspective, but you can, at least, define the outline or any necessary spot in perspective.

27. Applying the Equations to Draw a House (n = –30°)

This is the same house shown in Fig. 35. The same equations are used but with (n = –30° and F = 10).



$$X = \frac{F\,x}{z \sin n + y \cos n}$$

$$Y = \frac{F\,(z \cos n - y \sin n)}{z \sin n + y \cos n}$$

	x	y	z	X	Y		x	y	z	X	Y
a	-0.65	3.00	Q	-1.72	-1.42	o2	0.32	4.63	P	0.61	0.32
b	-3.13	6.55	Q	-4.57	1.81	p	0.51	4.77	R	0.99	1.22
c	0.64	3.90	Q	1.41	-0.19	q1	0.67	4.54	R	1.35	1.04
d	0.80	3.67	Q	1.84	-0.46	q2	0.67	4.54	S	1.43	2.09
e	2.14	4.61	Q	4.14	0.52	r	1.02	4.78	U	2.18	3.15
f	3.27	5.74	Q	5.32	1.36	s1	1.37	5.03	R	2.55	1.41
g1	-2.92	6.51	N	-4.02	0.61	s2	1.37	5.03	S	2.69	2.40
g2	-2.92	6.51	O	-3.97	0.39	t	2.17	5.93	R	3.53	1.96
g3	-2.92	6.51	P	-4.25	1.61	u	0.45	5.59	V	0.89	4.86
h1	-0.61	3.21	N	-1.38	-2.75	v1	-1.99	5.89	S	-3.24	1.83
h2	-0.61	3.21	O	-1.36	-3.04	v2	-1.99	5.89	S	-3.40	2.83
h3	-0.61	3.21	P	-1.52	-1.35	w	-1.85	5.69	T	-3.33	3.16
i1	0.68	4.11	N	1.31	-1.47	x1	-1.71	5.49	S	-2.95	1.60
i2	0.68	4.11	O	1.29	-1.74	x2	-1.71	5.49	S	-3.11	2.65
i3	0.68	4.11	P	1.42	-0.19	y	-1.09	5.93	S	-1.85	2.85
j1	0.83	3.89	O	1.63	-2.02	z	-1.04	6.26	T	-1.72	3.37
j2	0.83	3.89	P	1.80	-0.44	A	0.63	5.34	U	1.22	3.40
k1	1.92	4.65	O	3.34	-1.12	B	0.40	4.93	S	0.80	2.34
k2	1.92	4.65	P	3.65	0.34	C	-0.22	5.09	T	-0.44	2.89
l1	3.05	5.78	N	4.60	0.11	D	-0.27	4.76	S	-0.55	2.24
l2	3.05	5.78	O	4.54	-0.12	E1	-1.17	4.72	S	-2.28	1.05
l3	3.05	5.78	P	4.88	1.19	E2	-1.17	4.72	S	-2.42	2.21
m1	-0.45	4.09	N	-0.87	-1.49	F	-1.03	4.52	T	-2.27	2.57
m2	-0.45	4.09	P	-0.94	-0.21	G1	-0.89	4.32	S	-1.86	0.71
m3	-0.45	4.09	R	-0.99	0.63	G2	-0.89	4.32	S	-1.98	1.94
n1	-2.04	6.35	N	-2.86	0.51	H	-0.34	6.73	V	-0.56	5.01
n2	-2.04	6.35	P	-3.03	1.52	I	0.74	9.07	N	0.78	1.81
n3	-2.04	6.35	R	-3.13	2.18	J	-0.71	4.45	R	-1.46	0.96
o1	0.32	4.63	N	0.57	-0.89	K	-1.53	5.62	R	-2.60	1.79

(x) is the x-coordinate of a point (a) in the plan from the centre line of vision.
(y) is the y-coordinate of a point (a) in the plan from the Station Point (s).
(z) is the z-coordinate of a point (a) in the elevation from the Horizon Line (Eye Level).
(X) is the X-coordinate of a corresponding point in perspective from the centre line (Y-axis).
(Y) is the Y-coordinate of a corresponding point in perspective from the Focal Level (X-axis).
(F) is the Focal Length (a magnification factor - zoom in/out).
(n°) is the tilt angle of the camera.

Fig. 36. *Using the equations to draw a bird's eye- view of a house.*

Note: A permission is required if these equations are used in part of a computer programming to generate a computer 3-D model or any part of Computer Aided Design (CAD) that uses these equations to generate its 3-D model.

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