Difficulties Encountered by Junior High School on the Simplification of Rational Algebraic Expressions in Zambales, Philippines

Leonel Carnate Sanchez[1] and Teresita A. Falle[2]

[1,2]Graduate School, President Ramon Magsaysay State University, Iba, Zambales

Email: [1]leonel.sanchez@deped.gov.ph

***Abstract*—** Simplification of Rational Algebraic Expressions is apparently a challenging task for learners to accomplished. Misconceptions on the mathematical ideas and procedures on this process often leads to learners' errors. This study aimed to determine the difficulties encountered by Grade 8 Learners on the simplification of rational algebraic expressions using the Modular approach during pandemic in Zambales, Philippines. The study utilized the quantitative descriptive research design with item test on simplification of rational algebraic expressions and documentary analysis in gathering data from two hundred participants who were randomly selected. The students were assessed "Right Difficulty" in Simplifying Rational Algebraic Expressions. There is significant difference on the performance of the students between the two tests on rational algebraic expressions. There is positively light or weak relationship between the assessment for Simplifying Rational Algebraic Expressions and the level of academic performance. Based on the summary of the investigations conducted and the conclusions arrived at, the teacher may provide an intervention program through cooperative learning or differentiated learning to help and assist student with difficulty and struggle in Simplifying Rational Algebraic Expressions focusing on the encountered difficulties.

***Keywords*—** Algebra, Difficulties, Rational Algebraic Expressions, Simplifying.

I. INTRODUCTION

Students pays little attention on mathematical problems especially when fraction is involved. They perceived fraction as a difficult task to accomplished, with these reasoning they didn't pay more attention on their solutions. As a result, misconceptions and errors are done unintentionally. Students perceived Algebraic fractions as a challenge. The reason that Rational Algebraic expressions are subject to the laws of arithmetic functions makes it more difficult to learners. In addition, its numerator and denominator are polynomials. Algebraic fraction takes up the properties learnt about fractions and algebra. The computation of algebraic fractions is challenging to learners, possibly because they require the understanding of mathematical concepts like division, variable, equation, perfect squares, exponent, factorization, and rational numbers. Inadequacy in the understanding of these concepts leads to difficulties in solving algebraic fractions (Baidoo, 2019). Mhakure, Jacobs, & Julie (2014) argue that the learning of algebraic fractions or rational expressions is the most problematic area in school mathematics.

Numerous studies in some countries have been carried out on the way learners perform rational algebraic lessons. A study performed at the University of Western Cape, South Africa to Grade 9 learners on their ways of working with Rational Algebraic expression. The study reveals that learners use their instrumental understanding when they work with rational algebraic fractions, they lack adequate conceptual or relational understanding of what they do in each step as they work with rational algebraic fractions. With fractions that need factorization learners have difficulties in simplifying them (Maphini,2018). Research also conducted at South Africa on Grade 10 learners errors and misconceptions in simplifying Rational Algebraic Expressions on the goal of improving teaching and learning in their schools. The research showed that when grade 10 learners simplify rational algebraic expressions, they commit many errors and show various misconceptions (Makonye et al,2016). Most of the time these errors and misconceptions of learners when not addressed will lead to frequent practice and eventually makes learners skill on rational Algebraic Expressions more difficult to improve. Further study on these misconceptions and errors of learners when simplifying Algebraic Fractions at the Eastern Cape Province, South Africa. Learners' errors were analyzed to help teachers recognize learners' alternative conceptions or misconceptions so that they can help them learn mathematics adequately and effectively. Results indicated that conceptual, mathematical language, procedural and application errors hinder learners' appropriate understanding as well as the application of algebraic fractions. Notable among the four errors were conceptual and procedural errors that seemed to permeate attempts that learner made to simplify algebraic fractions (Baidoo, 2019). More explorations

were made on attempt to analyze the errors in the simplification of a rational expression of learners at Australia. The analysis of the errors across the whole data set produced three error categories (simple cancellation error, cancellation by subtraction error, cancellation by division of coefficients retaining the variable error) that were defined precisely. The results provide teachers with a snapshot of the difficulties students have in simplifying a rational expression of the type selected (Ruhl et al,2011).

The Philippines is still struggling in terms of students' performance in mathematics. As stated in the OECD 2018 PISA Country Note for the Philippines stated: "Fifteen-year-old students in the Philippines scored lower in reading, mathematics, and science than those in most of the countries and economies that participated in PISA 2018… In the 2018 tests, the Philippines ranked second to the last (Dominican Republic) in math and science (Villegas,2021). The performance of Filipinos in mathematics is notably low. Fifteen-year-old students belong in the junior high school and to address this problem on the performance of the county's learners, it is therefore a must to cater the difficulties encountered by the learners in mathematics to improve their performance. The subject itself has a wide scope so therefore this paper focuses only on a certain topic- the difficulties encountered by learners on the simplification of rational algebraic expressions.

In simplification of algebraic expressions, the errors learners make are related to their prior knowledge on common fractions and are strongly caused by learners depending more on cues rather than understanding (Figuaeras et al. 2008). This shows the importance of the two different understandings that are related to learning identified by Skemp (1976) (Relational understanding and instrumental understanding). According to Figuaeras et al. (2008), the reliance on cues when simplifying rational algebraic expressions is in line with the use of Skemps instrumental understanding, which requires learners to follow certain procedures without understanding as they simplify algebraic expressions.

As the learners do so, they retrieve wrong, incomplete, inappropriate, or flawed rules that lead them to make errors. In their research on errors learners commit when simplifying rational algebraic expressions, Figuaeras et al. (2008) and Mhakure et al. (2014), identified several errors and misconceptions learners make due to their prior knowledge and the dependence on instrumental understanding, which requires procedural understanding, and prior knowledge on simplifying common fractions. Such errors included the cancellation error, partial cancellation, like term error, linearization, and de-fractionalization and equationalization.

II. METHODOLOGY

This research is quantitative descriptive research design in nature because it would utilize numerical data in describing the level of competencies in math of grade 8 learners; the point of misconceptions/error in simplifying rational expressions; to perform simplifying rational algebraic expressions. A descriptive study was carefully designed to ensure complete description of the situation, making sure that there was minimum bias in the collection of data and reduce errors in the interpretation of the data. In the study, Grade 8 students from different public schools of zone 3 were involved. As this was definitely a very large population to handle, the researcher decided to work with a sample of the population. Using non-probability convenience sampling, 200 students were requested to participate in the study. Due to the current pandemic situation, questionnaires are distributed and included in their modules following the schedule of the distribution of their respective schools. The learners would work on the test items in the confinement of their homes under the supervision of their parents. Due to issue of confidentiality respondents are presented according to their sex, name of school and districts are not also mentioned in the results to respect their privacy.

This research study used questionnaires that includes exercises on simplifying rational algebraic expressions. To preserve the content validity, the content of the test was prepared by consulting the Learners material of the K-12 curriculum; Mathematics 8 as a basis. The exercises given consists of two parts. The exercise given to respondents consists of two parts. The first part of the exercise is a 10-item multiple-choice type of test. The items are consisting of rational expressions whose numerator and denominators are monomials and binomials. One the objective of the test is to identify the knowledge of the students on identifying the common monomial factor of the numerator and denominator of the given rational expression. It is a way to detect their ideas on how to perform cancellation, division, factorization of monomials and binomials in the numerator and denominator. The second part of the test is a 10-item simplifying of rational expression. Respondents are encouraged to show their whole process in solving the given rational expression. The objective of the second part of the test is to check their prior knowledge on factoring polynomials and on how do they apply these concepts in simplifying rational expressions. Special products are utilized in some of the given, if they can identify these special products, then

they can solve the items the easy way. The researcher was informed of the importance of being respectful and sensitive to the research participants that is why certain procedures was followed and respected. The researcher applied for the permission to conduct research to some public schools within the division of Zambales in the office of the school's division superintendent and applied to the school management of the schools to which research was conducted through their school heads. As the permission was approved, questionnaires were floated to randomly selected schools of the division of Zambales, zone 3, consisting of the secondary schools of the municipalities of Cabangan, San Felipe, San Narciso, and San Antonio.

Figure 1: *Map of Zambales Showing the Location of the Study*

Due to the current pandemic situation, questionnaires are given along with their modules following the schedule of the school's modular learning. The questionnaires are answered by the learners in the confinement of their homes. It is then retrieved along with their modules following the scheduled day. The answers and data that were collected were carefully checked, analyzed, and interpreted. Since the research study is quantitative. The following statistical tools was used: frequency distribution, percentage, mean, ANOVA and Pearson r. The software SPSS was also used to compute the needed data in this research[III].

RESULTS AND DISCUSSIONS

The study presents the gathered and processed data in a tabular form, analyzed, and provide interpretation of better understanding on the research problems.

Table 1: *Assessment of the Student-Respondents on the learning difficulties of grade 8 learners in Mathematics as to Simplifying Rational Algebraic Expressions by Factoring N=200.*

Level of Difficulty			
Item No.	**Number of Correct Answers**	**%**	**Interpretation**
1	133	66.50	Right Difficulty
2	87	43.50	Right Difficulty
3	135	67.50	Right Difficulty
4	103	51.50	Right Difficulty
5	105	52.50	Right Difficulty
6	111	55.50	Right Difficulty
7	88	44.00	Right Difficulty
8	93	46.50	Right Difficulty
9	88	44.00	Right Difficulty
10	86	43.00	Right Difficulty
	Mean	**51.45**	**Right Difficulty**

The data clearly demonstrate that out of ten (10) test items questions aims to check learner's understanding on the common factors present in the numerator and the denominator by simplifying rational algebraic expressions through application of simple factoring technique that were all assessed by the respondent's right difficulty. The items test was combination of monomial and simple binomials. Overall, the computed mean of the student performance for Simplifying Rational Algebraic Expressions by Factoring was 51.45 with qualitative interpretation of "Right Difficulty".

Table 2: *Assessment of the Student-Respondents on the learning difficulties of grade 8 learners in Mathematics as to Simplifying Rational Algebraic Expressions N=200*

Item No.	Number of Correct Answers	%	Interpretation
1	59	29.5	Right Difficulty
2	62	31.0	Right Difficulty
3	74	37.0	Right Difficulty
4	68	34.0	Right Difficulty
5	28	14.0	Difficult
6	53	26.5	Right Difficulty
7	59	29.5	Right Difficulty
8	62	31.0	Right Difficulty
9	74	37.0	Right Difficulty
10	68	34.0	Right Difficulty
	Mean	**30.30**	Right Difficulty

The data clearly demonstrate that out of ten (10) test items questions aims to check learner's understanding on the common factors present in the numerator and the denominator by simplifying rational algebraic expressions were assessed by the respondent's right difficulty and difficult respectively. The items test was combination of advance, higher and more complicated problems on algebraic expressions. The test items require students to show their complete process on simplifying the given items. For item 1, only 59 or 29.50% of the student-respondents got the correct answers while majority made a mistake on the ability of the learners to identify common factor in the numerator and the special product (difference of two squares) in the denominator. Majority of the learners failed to identify

the common factors in the numerator and the failed to factor the denominator. There is also an evident problem on cancellation.

For item 2, only 62 or 31.00% of the student-respondents got correct answers while majority made error on the problem. The figure shows the work of a learner out of the 110 learners who failed to simplify item B. The learner successfully identified the common factor in the numerator but failed to simplify correctly. The learner also failed to factor the special product in the denominator. Incorrect cancellation is also shown on the solution

For item 3, only 74 or 37.00% of the student-respondents got correct answers while majority made to choose the right answer remaining learners did not attempt to answer the question. This item aims to know the learners' ability to identify and factor a trinomial in the numerator. The denominator still aims to test their ability on difference of two squares.

For item 4, there were 68 or 34.00% of the student-respondents got the correct answer while majority made error in solving the algebraic expression. This item aims to know the skill of the learners to decide what to do if grouping symbols are present in an expression. The figure shows the solution of one of the learners.

The learners successfully applied the distributive property to evaluate the expression but forgot to remove the parenthesis after the evaluation. The learner got the correct product in multiplying the first term in the expression inside the parenthesis but failed in the second term. That is why the answer a2 + 5 must be a2 + 5a. The learner assume it is a2 + 5 in order to cancel out a2 + 5 in the denominator which leads to an incorrect answer.

For item 5, only 28 or 14.00% of the student-respondents got correct answer and considered "Difficult" on the item. This item aims to know the skill of the learners to decide what to do if grouping symbols are present in an expression. The figure shows the solution of one of the learners.

The learners successfully applied the distributive property to evaluate the expression but forgot to remove the parenthesis after the evaluation. The learner got the correct product in multiplying the first term in the expression inside the parenthesis but failed in the second term. That is why the answer a2 + 5 must be a22+ 5a. The learner assume it is a2 + 5 in order to cancel out a2 + 5 in the denominator which leads to an incorrect answer. Only 30% of the learners got the correct answer.

For item 6, there were 53 or 26.50% of the student-respondents got the correct answer while still majority made the error in solving the problem. This item aims to know the knowledge of the learners on identifying and factoring difference of two cubes in the numerator and factoring a trinomial in the denominator. The figure shows the solution of one of the 84 learners who attempted to answer item F. The learner failed to factor x3 -1 correctly but managed to get the correct factors of the trinomial in the denominator. Although the cancellation is done correctly the factors of the numerator is incorrect that leads the answer to be incorrect. 31% of the learners got correct answers and the remaining 54 did not attempt to answer the question.

For item 7, only 53 or 26.5% of the 200 student-respondents got the correct answers and majority made a mistake on solving for the problem. Item #6 requires the learners to show the factored form of the trinomial in the numerator to apply cancellation technique. Among the 200 learners there are 74 who answered correctly. 69 attempted to answer the question and 57 leave their answers blank. The figure shows of the learner's solution in item 6. Improper application of cancellation technique was applied. The learner failed to factor the trinomial

For item 8, only 59 or 29.50% of the 200 student-respondents got the correct answer. Item 7 aims to know the ability of the learners to factor difference of two squares in the denominator. 68 of them answered correctly, 77 attempted but failed and the rest of the learners leave their answers blank. The figure shows the incorrect factoring technique. Simplifying the algebraic fraction would be easier if the learner factored x2 – 16 into (x + 4) (x - 4). Some of the learners did the right thing in factoring the denominator but made a mistake in transforming their answer into a non-fraction. Instead 1/x +4 they answered x + 4.

For item 9, only 74 or 37.00% of the student-respondents made correct answers while majority commit errors in solving the problem. Item 9 aim to know the ability of the learner on how to factor trinomials in the form ax2 + bx + c in the numerator and denominator to simplify the rational expression. The learner failed to apply factoring technique and incorrect cancellation is also applied.

For item 10, out of 200 student-respondents, only 68 or 34.00% got the correct answer. Majority commit errors on applying factoring on trinomials and division of negatives are the skills needed to simplify the given rational expression. 53 learners got correct answers, 90 attempted but failed and 57 chose not to answer the

question, the solution of one of the learners shows no factoring technique was applied. Incorrect application of cancellation technique is also evident. Overall, the computed mean/percentage was 30.30 interpreted as "Right Difficulty". Learners were found to have the following errors that leads them to have difficulties as to the failure to apply factoring technique; incorrect cancelling/ Partial Cancelling; poor in identifying the common factor (greatest common factor); and lack of ability to apply defractionalisation.

Table 3: *t-test determine differences between the two tests on rational algebraic expressions.*

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Multiple Choice	200	1.4855	.24829	.01756
Simplifying	200	1.6695	.25543	.01806

One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Multiple Choice	84.611	199	.000	1.48550	1.4509	1.5201
Simplifying	92.432	199	.000	1.66950	1.6339	1.7051

There is significant difference on the performance of the students on the two tests on rational algebraic expressions manifested by the computed Sig. (2-tailed) value of 0.000 which is less than (<) 5% Significance Level, therefore the null hypothesis is rejected.

The data clearly demonstrate on the difficulty of the students encountered on solving problems on simplifying rational algebraic expressions considering that in factoring problems only involve monomial and simple binomial problems compare to the latter where it deals with higher, advance and more complicated problems on algebraic expressions.

Table 4:

Analysis of Variance to test difference on the test item for Simplifying Rational Algebraic Expressions by Factoring

Groups	*Count*	*Sum*	*Average*	*Variance*
Item Test #1	200	133	0.665	0.223894
Item Test #2	200	87	0.435	0.24701
Item Test #3	200	135	0.675	0.220477
Item Test #4	200	103	0.515	0.25103
Item Test #5	200	105	0.525	0.250628
Item Test #6	200	111	0.555	0.248216
Item Test #7	200	88	0.44	0.247638
Item Test #8	200	93	0.465	0.250025
Item Test #9	200	88	0.44	0.247638
Item Test #10	200	86	0.43	0.246332

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	15.4345	9	1.714944	**7.049003**	4.29E-10	**1.884576**
Within Groups	484.145	1990	0.243289			
Total	499.5795	1999				

Decision: Reject Null Hypothesis (There is Significant Difference)

Table 4 shows the Analysis of Variance to test difference on the test item for Simplifying Rational Algebraic Expressions by Factoring.

There is significant difference on the test item for Simplifying Rational Algebraic Expressions by Factoring manifested on the computed F –value of 7.049003 which is greater than the F-critical value of 1.884576, therefore the null hypothesis is rejected.

The test items denote differentiated difficulty where majority committed errors in solving test item for Simplifying Rational Algebraic Expressions by

Factoring despite on giving monomial and basic binomial problems.

Table 5:

Analysis of Variance to test difference on the test item for Simplifying Rational Algebraic Expressions

Groups	*Count*	*Sum*	*Average*	*Variance*
Simplify1	200	88	0.44	0.247638
Simplify2	200	90	0.45	0.248744
Simplify3	200	74	0.37	0.234271
Simplify4	200	65	0.325	0.220477
Simplify5	200	59	0.295	0.20902
Simplify6	200	62	0.31	0.214975
Simplify7	200	74	0.37	0.234271
Simplify8	200	68	0.34	0.225528
Simplify9	200	28	0.14	0.121005
Simplify10	200	53	0.265	0.195754

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	14.3545	9	1.594944	**7.412542**	1.04E-10	**1.884576**
Within Groups	428.185	1990	0.215168			
Total	442.5395	1999				

Decision: Reject Null Hypothesis (There is Significant Difference)

Table 5 shows the Analysis of Variance to test difference on the test item for Simplifying Rational Algebraic Expressions.

There is significant difference on the test item for Simplifying Rational Algebraic Expressions manifested on the computed F –value of 7.412542 which is greater than the F-critical value of 1.884576, therefore the null hypothesis is rejected.

Table 6:

Pearson Product Moment Coefficient of Correlation to test relationship between the level of academic performance and the assessment test for Simplifying Rational Algebraic Expressions by Factoring

Sources of Correlations		Academic Performance	Multiple Choice
Academic Performance	Pearson Correlation	1	0.300**
	Sig. (2-tailed)		0.000
	N	200	200
Multiple Choice	Pearson Correlation	0.300**	1
	Sig. (2-tailed)	0.000	
	N	200	200

**. Correlation is significant at the 0.01 level (2-tailed).

Table 6 shows the Pearson Product Moment Coefficient of Correlation to test relationship between the level of academic performance and the assessment test for Simplifying Rational Algebraic Expressions by Factoring. There is positively light or weak relationship between the assessment test for Simplifying Rational Algebraic Expressions by Factoring and the level of

academic performance and the manifested on the computed Pearson r- value of 0.300**. The computed Sig. (2-tailed) value of 0.000 which is lower than 5% significance level, therefore the null hypothesis is rejected, hence there is significant relationship.

Table 7:

Pearson Product Moment Coefficient of Correlation to test relationship between the level of academic performance and the assessment test for Simplifying Rational Algebraic Expressions

Sources of Correlations		Academic Performance	Simplifying
Academic Performance	Pearson Correlation	1	0.251**
	Sig. (2-tailed)		0.000
	N	200	200
Simplifying	Pearson Correlation	0.251**	1
	Sig. (2-tailed)	0.000	
	N	200	200

**. Correlation is significant at the 0.01 level (2-tailed).

Table 7 shows the Pearson Product Moment Coefficient of Correlation to test relationship between the level of academic performance and the assessment test for Simplifying Rational Algebraic Expressions. There is positively light or weak relationship between the assessment test for Simplifying Rational Algebraic Expressions and the level of academic performance manifested on the computed Pearson r- value of -0.251**. The computed Sig. (2-tailed) value of 0.000 which is lower than 5% significance level, therefore the null hypothesis is rejected, hence there is significant relationship.

The data further implies that as the result of the academic performance is negatively influence or associated by the results of the students' performance in the assessment test for Simplifying Rational Algebraic Expressions. This further explains that as the result of the assessment is slightly increasing, the level of academic performance is also slightly increasing.

IV. CONCLUSION

Based on the summary of the investigations conducted, the researcher has arrived at the conclusions that:

1. The students were rated or obtained a "Very Satisfactory" level of academic performance.
2. The students were assessed "Right Difficulty" in Simplifying Rational Algebraic Expressions by Factoring and Simplifying Rational Algebraic Expressions.
3. There is significant difference on the performance of the students between the two tests on rational algebraic expressions.
4. There is positively light or weak relationship between the assessment for Simplifying Rational Algebraic Expressions by Factoring and the level of academic performance and there is positively light or weak relationship between the assessment test for Simplifying Rational Algebraic Expressions and the level of academic performance.

V. RECOMMENDATIONS

Based on the summary of the investigations conducted and the conclusions arrived at, the researcher has offered the following recommendations based on salient findings obtained in the study:

1. The teacher may provide an intervention program through cooperative learning or differentiated learning to help and assist student with difficulty and struggle in Simplifying Rational Algebraic Expressions focus on the failure to apply factoring technique; incorrect cancelling/ Partial Cancelling; poor in identifying the common factor (greatest common factor); and lack of ability to apply defractionalisation
2. The teacher may review the lesson focus on the identified errors committed by the students to ensure mastery as to the failure to apply factoring technique; incorrect cancelling/ Partial Cancelling; poor in identifying the common factor (greatest common factor); and lack of ability to apply defractionalisation.
3. Conduct conference with parents and encourage to help and assist child with struggle and difficulty in learning Mathematics.
4. To conduct a parallel or similar study with in-depth and wider in scope to validate the findings obtained in the study.

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