Controlling Biscuit Raw Material Inventory Using a Probabilistic Model Q-Back Order and Q-Lost Sales

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***Abstract*—** Inventory management in a company is an important factor in determining a company's competitive success. PT. Siantar Top Tbk is a company that produces snacks, with biscuits being one of its main products. There is always a fluctuating and unexpected demand for biscuit products at this company, resulting in the company's inability to meet the needs of products based on orders. This is due to a shortage of raw materials, which is the main factor in the production of biscuits. To achieve optimum inventory costs, it is necessary to formulate a formula related to how inventory is in accordance with the probabilistic pattern of raw material demand. The results of this study, which aims to determine the probabilistic inventory of biscuit raw materials for the Q model and then compare the total cost of biscuit raw material inventory for the Q-back order and Q-lost sales models, will be explained in this paper.

***Keywords*—** Inventory, Probabilistic Model, Q Model, Q-back order, Q-lost sales.

I. INTRODUCTION

Inventory control is a crucial issue [1-3]. The amount of inventory will influence or determine how smoothly the production process goes as well as the organization's efficacy and efficiency [4]. Not only can delays or low inventory affect the production process, but they will also prevent a company from taking advantage of opportunities to boost revenues. Depending on the volume of production, the plant's capacity, and the manufacturing method, a corporation may need a varied amount or level of inventory [5,6].

Inventory management is the supervision of non-capital assets and inventories of goods. Supply chain management also includes inventory management. Finding a balance between customer service and investment is the aim of inventory management [7]. There has been research on inventory management systems that take into account things like co-orders, expiration dates, and all unit discounts, for instance [5,6,7,8, and 9]. [10] has conducted research on inventory control models that take into account restricted warehouse capacity for deterministic demand, while [11] and [12] have done the same for probabilistic demand. It is believed that the discovery model with probabilistic demand is more indicative of the actual situation [13].

In this case, companies typically construct safety stock, which is computed directly from estimates [14,15], to deal with probabilistic demand. This statement is reinforced by several research findings [16–18], which show that by employing inventory management, companies can boost employee productivity and reduce costs [19]. Researchers [21–22] conducted studies to back up this statement. The company produces a variety of biscuits, including Go Riorio, Go Tiramisu, Go Nanas, Go Potato, and others. The company's biscuit goods are marketed to all market segments because they are reasonably priced when compared to other biscuit products of a similar kind, have a tasty texture that appeals to consumers, and have a long shelf life.

There are 12 different types of raw materials needed to make biscuits. These supplies are divided into three categories:

1. The main raw materials consist of wheat flour, vegetable oil, sugar, cocoa powder, milk powder,
2. Additional ingredients include vanilla creamer, salt, baking powder, and creamy milk.
3. Auxiliary raw materials include mono and diglycerides, emulsifiers, dextrose, and maltose.

Due to fluctuations in product demand at the company, which fluctuated and unexpectedly resulted in product demand needs not being met according to order, due to shortages that occurred in raw materials, which are the main raw materials in biscuit products.

II. LITERATURE REVIEW

The standard deviation will be used to represent the highest demand in the probabilistic inventory model even though it is uncertain but not probabilistic. As a result, estimating the size of the safety recommendations

will be difficult. The simplest method for resolving the issue is to use nuclear probabilistic thinking, which adds a safety recommendation (ss) to the static deterministic inventory system to anticipate and reduce demand fluctuations [16].

A. Model Q with Back Order

The model formulations and solutions that follow are only valid when backorders are used to handle inventory shortages. In this case, the user is willing to wait for the desired item to become available in the warehouse, and the management will attempt to meet the unmet demand by placing an emergency order.

1. Model formulation and solutions

The results obtained from the substitution in the OT If the inventory shortage is treated by way of back order, the following will be obtained:

$$O_T = O_b + \mathrm{O_p} + \mathrm{O_k}$$

$$\boldsymbol{O_T} = \mathrm{Dp} + \frac{AD}{\boldsymbol{q_0}} + \mathrm{h}\left(\frac{1}{2}q_0 + r - D_L\right) + \left(\frac{1}{2}q_0 + C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty}(x-r)f(x)dx\right.$$

To find the value of the optimal decision variables, *r* and *ss* are obtained by using the optimization principle, namely by utilizing the convexity of O_T to q_0 and *r*. Thus, the condition for minimum O_T is:

a. $\frac{\partial O_T}{\partial q_0} = 0$

$$\frac{AD}{\boldsymbol{q_0}} + \frac{1}{2}\mathrm{h} - C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty}(x-r)f(x)dx = 0$$

$$\mathrm{h}q_0 = 2\mathrm{AD} + 2C_uD\int_r^{\infty}(x-r)f(x)dx$$

$$q_0{}^* = \sqrt{\frac{2D\,(A + C_u\int_r^{\infty}(x-r)f(x)dx)}{h}}$$

b. $\frac{\partial \mathrm{O_T}}{\partial \boldsymbol{q_0}} = 0\mathrm{h} - C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty} f(x)dx = 0$

$$\int_r^{\infty}\boldsymbol{f(x)dx} = \frac{\boldsymbol{hq_0}^*}{\boldsymbol{C_{uD}}}$$

Thus, the probability of an inventory shortage can be expressed as:

$$\boldsymbol{\alpha} = \frac{\boldsymbol{hq_0}^*}{\boldsymbol{C_uD}}$$

2. Hadley-Within Method Solution

To determine the value ofq_0* andr_0*, search iteratively:

a. Calculate the order lot q_{01}* with Wilson's formula.

b. Based on the value of q_{01}* obtained, it can be found the magnitude of the possibility of a lack of inventory.

c. By knowing the r_1* obtained, the value of q_{02}* can be calculated.

d. Recalculate the value of $\boldsymbol{\alpha} = \frac{\boldsymbol{hq_{02}}^*}{\boldsymbol{C_uD}}$ and value $\boldsymbol{r_2^*}$ again.

e. If price $\boldsymbol{r_2^*}$ relative to $\boldsymbol{r_1^*}$ iteration is complete and will be obtained $\boldsymbol{r_1^*} = \boldsymbol{r_2^*}$dan $q_0{}^* = q_{02}{}^*$,otherwise return to step c by substituting values for $\boldsymbol{r_1^*} = \boldsymbol{r_2^*}$ dan $q_{01}{}^* = q_{02}{}^*$

f. For safety stock:

$$\mathrm{Ss} = Z\alpha s_L$$

g. Expected total cost per year:

$$\mathrm{O_T} = \mathrm{Dp} + \frac{AD}{\boldsymbol{q_0}} + \mathrm{h}\left(\frac{q_0 + r - D_L}{2}\right) + C_u\left(\frac{D}{\boldsymbol{q_0}}\right)xN$$

B. Model Q with Lost Sales

Only if the inventory shortfall is considered in terms of lost sales are the model formulations and solutions that follow applicable. In this case, the user does not want to wait for the requested item to be available in the warehouse. Users will search elsewhere for their needs.

1. Model formulation and solution

It will be reached by treating the outcomes of substituting in OT with a scarcity of inventory as lost sales.

$$O_T = \mathrm{Dp} + \frac{AD}{\boldsymbol{q_0}} + \mathrm{h}\left(\frac{1}{2}q_0 + r - D_L\right) + \left(\frac{1}{2}q_0 + C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty}(x-r)f(x)dx\right.$$

The optimal decision variable will be obtained using the optimization principle in the same manner as in the back-order case.

2. Hadley-Within Method Solution

To determine the value of $\boldsymbol{q_0}^*$dan $\boldsymbol{r_0}$* search iteratively.

III. RESEARCH METHODS

The Q model is used to solve inventory problems as follows:

1. Modeling Using the Statistical Program for Social Science (SPSS) Version 16: The demand data is modeled in order to determine the results of the modeling. The test was carried out using the Statistical Program for Social Science (SPSS) version 16.
2. Select the smallest MSE and do a forecast: To find out the number of requests in the future, forecasting

calculations are carried out using forecasting methods that are in accordance with the data pattern. The method chosen for forecasting is the forecasting method that has the smallest MSE.The linear trend forecasting method was chosen in this case because it has the lowest MSE.

3. Calculation of inventory policy using the Q model: In this study, the Hadley-Within method with the Wilson formula was used to solve the problem of the probabilistic inventory policy Q model. This method yields the economic number of orders, the point of raw material reordering, and the optimal total cost of inventory for a year. This study also considers the condition of inventory shortages through the use of back orders and lost sales mechanisms.

IV. RESULTS AND DISCUSSION

A. Collecting Data

1. Biscuit demand data for 2018

The demand data used in the study is monthly demand data for Go Riorio Biscuits for one year in 2018, as shown in the following table 1.

Table 1:*Data on Monthly Product Demand*

No	Month	Demand (box)
1	January	56,064
2	February	65,043
3	March	54,674
4	April	55,744
5	May	61,345
6	June	62,769
7	July	53,126
8	August	60,876
9	September	59,765
10	October	58,766
11	November	63,654
12	December	65,657
Total		**717,483**

2. The biscuit's raw material composition

There are five raw materials used to make biscuits, with quantities as shown in the following table 2.

Table 2: *Raw Material Quantity Data*

No	Raw Material	Quantity (gr)
1	Wheat flour	14
2	Vegetable oil	59
3	Sugar	3
4	Chocolate powder	2
5	Milk powder	4

1). The price of raw materials and the lead time
The following table 3 shows the price and lead time data for the five raw materials used to make Go Riorio biscuit products.

Table 3: *The Price of Raw Materials and The Lead Time Data*

Raw Material	Price (IDR)	Lead Time (day)
Flour	5.000/Kg	7
Vegetable oil	9.000/L	7
Sugar	10.000/Kg	7
Chocolate Powder	52.000/Kg	7
Milk powder	35.000/Kg	7

3. Ordering cost data

Ordering costs are all costs incurred related to ordering materials. The ordering cost for each order can be seen in the following table 4.

Table 4: *Ordering Cost Data*

No	Item	Cost (IDR/order)
1	Phone Charges	44,792
2	Administrative costs	38,542
3	Transportation costs	31,250
Total		**114,584**

4. Storage cost data

The costs incurred are related to the storage of materials before they are used for production. The costs incurred are as shown in the following table.

Table 5: *Storage Cost Data*

No	Item	Cost (IDR/year)
1	Electricity	5,280,000
2	Administrative costs	850,000
Total		**6,130,000**

A. Forecasting.

The forecasting model uses SPSS Version 16 software. There are four methods of forecasting, namely linear, logarithmic, square and exponential. Of the four methods tried, the one with the smallest MSE value is the linear trend method. For this reason, this method is used to forecast demand for the following year. Forecasting results in one year are 114,797,280 packs, which have been converted from box to pack, where one box contains 160 packs.

Based on the results of demand forecasting, it can be seen the need for raw materials in one year, as shown in the following table 6.

Table 6: *The Need for Raw Materials*

No	Raw Material	Quantity
1	Wheat flour (Kg)	2,084,221.44
2	Vegetable oil (L)	8,783,504.64
3	Sugar (Kg)	446,618.88
4	Chocolate powder (Kg)	297,745.92
5	Milk powder (Kg)	595,491.84

B. Calculation of the Raw Material Inventory Probabilistic Model with Q-Back Order

The steps for calculating the inventory of wheat flour raw materials using the Q model with backorders are as follows.

The Hadley-Within method, the Wilson formula, and iterations will be used to determine the value of the order lot size $q_0{}^*$and r_1^* the back order.

Calculations performed and the results obtained in iteration 1:

a. Counting lot orders $q_{01}{}^*$ = 23,037 Kg
b. Calculate the value of reordering by first knowing the magnitude of the possibility of a shortage of inventory. Reorder point value (r_1*) = 487,912 Kg.
c. Recalculate the lot order value and the results obtained are q_{02}* = 29,244 Kg.
d. After getting the lot order value, it will be known that the reorder point is r_2* = 487,746 Kg.

The order point value from the calculation of the Q-back order model is r_1* = 487,912 Kg and r_2* = 487,746 Kg.

Because they both show the same results, there is no need to proceed to the next iteration, and the calculation of the Q-back order model is concluded with the following conclusions:

1). Optimum order quantity
$q_0{}^* = q_{02}{}^* =$ 29,244 Kg.
2). Reorder point
$\boldsymbol{r_1^* = r_2^*} =$ 487,746 Kg.
3). Safety stock for back order

$$Ss = Z\alpha \,.\, s_L$$

= 8,374.82 Kg
4). Total expected cost for a year using the Q-back order model:

$$\boldsymbol{O_T} = \text{Dp} + \frac{AD}{\boldsymbol{q_0}} + \text{h}\left(\frac{1}{2}q_0 + r - D_L\right) + \left(\frac{1}{2}q_0 + C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty}(x-r)f(x)dx\right)$$

O_T = IDR 65,423,019

Likewise, for other raw materials, can be done in the same way as in the calculation of wheat flour. The complete results can be seen in the following table 7 below.

Table 7: *Calculation Results for Model Q-Back Order*

Raw Material	q	r	SS	O_T (IDR)
Wheat Flour	29,244	487,746	8,374.82	65,423,019
Vegetable Oil	65,536	2,059,012	38,805.71	495,022,937
Sugar	8,853	104,187	1,464.25	27,999,336
Cocoa Powder	2,919	69,732	1,249.97	97,022,297
Milk Powder	6,574	139,404	2,440.42	130,585,515

C. ***Calculation of the Raw Material Inventory Probabilistic Model with Q-Lost Sales***

The steps for calculating the inventory of wheat flour raw materials using the Q model with lost sales are as follows.

The Hadley-Within method, the Wilson formula, and iterations will be used to determine the value of the order lot size $q_0{}^*$and r_1^* the back sales.

Calculations performed and the results obtained in iteration 1:

a. Counting lot orders $q_{01}{}^*$ = 23,037 Kg
b. Calculate the value of reordering by first knowing the magnitude of the possibility of a shortage of inventory. Reorder point value (r_1*) = 487,954Kg.
c. Recalculate the lot order value and the results obtained are q_{02}* = 29,232Kg.
d. After getting the lot order value, it will be known that the reorder point is r_2* = 487,537 Kg.

The order point value from the calculation of the Q-lost sales model is r_1* = 487,954 Kg and r_2* = 487,537 Kg. Because they both show the same results, there is no need to proceed to the next iteration, and the calculation of the Q-lost sales model is concluded with the following conclusions:

1). Optimum order quantity
$q_0{}^* = q_{02}{}^* =$29,232Kg.
2). Reorder point
$\boldsymbol{r_1^* = r_2^*} =$ 487,537Kg.
3). Safety stock for lost sales

$$Ss = Z\alpha \,.\, s_L$$

= 81,66.49 Kg
4). Total expected cost for a year using the Q-lost sales model:

$$\boldsymbol{O_T} = \text{Dp} + \frac{AD}{\boldsymbol{q_0}} + \text{h}\left(\frac{1}{2}q_0 + r - D_L\right) + \left(\frac{1}{2}q_0 + C_u\frac{D}{\boldsymbol{q_0}}\int_r^{\infty}(x-r)f(x)dx\right)$$

O_T = IDR65.325.844

Likewise, for other raw materials, can be done in the same way as in the calculation of wheat flour. The complete results can be seen in the following table 8 below.

Table 7: *Calculation Results for Model Q-Lost Sales*

Raw Material	q	r	SS	O_T (IDR)
Wheat Flour	29,232	487,537	81,66.49	65,325,844
Vegetable Oil	65,536	2,059,187	38,981.30	495,023,177
Sugar	8,792	104,472	1,749.96	28,020,863
Cocoa Powder	3,635	69,684	1,202.35	97,376,063
Milk Powder	6,413	139,427	2,464	130,586,071

V. CONCLUSION

By using biscuit demand data, the number of biscuits demanded for the following year can be calculated and processed using trend methods, which include linear trends, logarithmic trends, quadratic trends, and exponential trends. After forecasting using these four methods, the best method is obtained, namely Linear Regression with the lowest Mean Square Error (MSE). In one year, the predicted yield is 717,483 boxes, or 114,797,280 packs.

The optimal value for the number of orders, reorder points, and safety stock is given by the probabilistic inventory model Q with back orders and lost sales. Based on these findings, the company can determine when to order raw materials to avoid production delays.

The lowest total cost is IDR 65,423,019/year for wheat flour; IDR 495,022,937/year for vegetable oil; IDR 27,999,336/year for sugar; IDR 97.376.063/year for chocolate powder; and IDR 130,585,515/year for milk powder when using the Q model with a back order.

The lowest total cost is IDR 65.325.844/year for wheat flour; IDR 495.023.177/year for vegetable oil; IDR 28.020.863/year for sugar; IDR 97,022,297/year for chocolate powder; and IDR 130.586.071/year for milk powder when using the Q model with lost sales.

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