Exploring the Transition of the Learning Content of Fraction from Elementary to Secondary Level

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***Abstract*—** Mathematics learning is well understood to be the process that must be considered at every level of education through a very comprehensive manner. Looking through, there are three interconnected elements in mathematics learning: teachers, students and materials/contents that generate the outcome of this learning. This study sought to investigate the learning content in fractions that transcend from elementary to secondary level. It utilized the traditions of a descriptive phenomenological research approach using pure qualitative data to give answers to the prevailing research questions. The learning contents in fraction transcend from elementary to secondary level following the framework of the CPA Approach with the sequence, namely: 1) Concrete and action-based representation using manipulative skills; 2) Pictorial representations of concrete objects; and 3) Abstract Notations using Mathematical Symbols. The learning contents in fraction scaffold using strategies, namely: 1) conduct of diagnostic assessment among learners; 2) review of learner's previous knowledge; 3) use of real, touchable, manipulative and concrete objects; 3) Peer support or tutorial approach learning; 4) learner-made learning materials; 5) contextualizing and localizing learning modules; and 6) providing learning support facilities. The learning contents in fraction transcend from elementary to secondary level according to the framework and principles of CPA Approach. There are varied strategies that teachers use to scaffold the learning content in fraction that transcend from elementary to secondary level. Elementary and Secondary teachers may become attentive and observant about the principles of their teaching to ensure that different empirical approaches such as the CPA Approach may appropriately and accordingly apply in their instructional practices especially when teaching fraction to fulfil the needed skills of learners in preparation to the next level of learning contents in fraction and other mathematical concepts. Varied and purposeful scaffolding strategies provided in the proposed handbook may be utilized to apply CPA in teaching fraction. The proposed teacher's handbook entitled Concrete-Pictorial-Abstract Approach (CPA Approach) to Teach Fraction & Other Mathematical Concepts – A Teacher's Handbook may be utilize to enhance the teaching skills of mathematics teachers in elementary and secondary. Action research along the utilization of CPA Approach as intervention may be conducted.

***Keywords*—** Mathematics, Transition of the Learning Contents of Fraction; Strategies, CPA Approach, Descriptive Phenomenological

INTRODUCTION

Improving mathematics learning time after time is given an ultimate attention by every education practitioner. This goal is true not only worldwide but most especially in the Philippines that right now is undergoing a radical change in its curriculum. Mathematics as a tool subject, is overarching other tool subjects as well. It does not exist alone, on its own but interrelated with other tool subjects such as Sciences and Languages. Mathematics learning is well understood to be the process that must be considered at every level of education through a very comprehensive manner. Looking through, there are three interconnected elements in mathematics learning: teachers, students and materials/contents that generate the outcome of this learning. It is essential that these three are well considered in the effort of improving mathematics learning.

According to Machmud (2011), in the process of teaching and learning of Mathematics, teachers play a role to plan, design process, provide context and create a learning environment such that the material presented can be understood by students, develop their schemata, also giving them opportunity to construct their understanding, interacting with each other in expressing ideas so that it will produce optimal learning outcomes. Teachers are the designers of the teaching process itself with goals of education to keep in mind. Such effort will direct students to gain that mathematical thinking and high order mathematical thinking. Sumarmo (2008) posits that mathematical thinking can be defined as conduct or mathematical process (doing math) or mathematical tasks.

The twin goals of Mathematics in the basic education level as indicated in the K to 12 Basic Education Curriculum (Department of Education, 2016) are

Critical Thinking and Problem Solving. According to this existing curriculum for Mathematics, critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. While mathematical problem solving is finding a way around a difficulty, around an obstacle, and finding a solution to a problem that is unknown. These two goals are to be achieved with organized and rigorous curriculum content, a well-defined set of high-level skills and processes, desirable values and attitudes, and appropriate tools, taking into account the different contexts of Filipino learners.

Another feature of the said curriculum is anchored on the principles of a spiral progression approach. The K to 12 Basic Education was implemented in 2011 through the power instituted in the Republic Act No. 10533, otherwise known as the "Enhanced Basic Education Act of 2001". It stated that the curriculum shall use the spiral progression approach to ensure mastery of knowledge and skills after each level and the overall design of Grades 1 to 10 curricula follows the spiral approach across subjects by building on the same concepts developed in increasing complexity and sophistication starting from grade school. This means that teachers should observe in their process of mathematics teaching the spiral progression as the guiding principle. This considers how learning content from a previous level is connected to the next level for learners to be ready to learn contents for that present level.

With this principle, the teaching process in mathematics has vertical articulation or seamless progression and the horizontal articulation of competencies. Samala (2018) shared that vertical articulation serves as a bridge of knowledge from one lesson to the next, across a program of study. It develops skills and knowledge which are reinforced as other elements are used in the study. While, horizontal articulation integrates the skills and knowledge across different disciplines. It means that what has been studied in one specific course or area is in line with the other.

Meanwhile, in the conduct of this present study, the researcher would like to explore how learning contents in fraction transcend beginning from elementary to secondary level. Given the spiral approach, the vertical articulation/seamless progression and the horizontal articulation, teachers as teaching process designers have come up with different teaching strategies so that they can make their learners instill from the simplest to the most sophisticated concepts and principles of fraction. This way, the picture of the transition of these concepts at the learners' respective level can be viewed through the teaching process from the teaching ground itself. It also would like to look into the scaffolding activities that teachers designed to support their students' learning of fraction at different progression of the competencies and contents.

Wiese and Koedinger (2015) disclosed that a common lament in mathematics education is that students often execute procedures without understanding: without connecting procedural steps to their underlying concepts and without connecting symbols to their 'real-world' referents. According to Sowder and Wearne (2006), fractions are one of the most important topics students need to understand in order to be successful in algebra and beyond. But, it is an area in which students struggle. NAEP test results have consistently shown that students have a weak understanding of fraction concepts.

Later, this skill deficiency is translated as difficulties with fraction computation, decimal and percent concepts and the use of fractions in other content areas, particularly algebra (Bailey, Hoard, Nugent, & Geary, 2012). Therefore, it is absolutely critical that fractions are taught well, interestingly and importantly.

This is also true laments of teachers while their real concern is how to make their students learn the concepts of the fraction lessons presented to them. Many students memorize the procedure without giving time to understand the concepts underlying the procedures. In like manner, students would easily forget what they learn if it is only the procedures that they will learn. They will have limited and short-term learning because they did not learn the concept.

On the other hand, if students fully understood the content on fraction, they would be able to come up with mathematical solution creatively on their own. This kind of learning may ensure a life-long learning towards the twin goal of the curriculum: Critical Thinking and Problem Solving.

This study sought to investigate the learning content in fractions that transcend from elementary to secondary level. Specifically, it answered the following questions:

1. How are the learning contents in fraction transcend from elementary to secondary level?
2. How do teacher scaffold the learning contents in fraction that transcend from elementary to secondary level?

MATERIALS AND METHOD

It utilized the traditions of a descriptive phenomenological research approach. It utilized a pure qualitative data to give answers to the prevailing research questions.

To gather the needed qualitative data, two separate sessions of focus group discussion (FGD thereafter) were employed. Transcription of the proceedings of these FGDs were done as basis for the collecting the codes and categories to achieve the goals of this research through directed content analysis. Documentary analysis, pictorial documentation, interview and observations were also used to confirm the gathered data from FGD.

FGDs were composed of mathematics teachers from Grade 1 to Grade 6 (elementary levels), and Grade 7 to Grade 10 (junior secondary levels) from Magallanes District of Sorsogon Province Division. There were seven elementary teachers who attended the first session of FGD, and six secondary teachers who attended the second session. Establishing codes and categorize on the informants' responses were done to achieve the goal of this present research.

Participants

There were 13 mathematics teachers who participated in the FGDs conducted. They all belonged to the public schools of Magallanes District in Sorsogon Province Division. Seven of them were teaching in the elementary levels and six of them were teaching in the junior secondary levels with three to 26 years of teaching experience.

Of the seven elementary mathematics teachers, one was a Grade 1 teacher with 26 years of teaching experience, two Grade 2 teachers with five and seven years, two Grade 3 teachers with six and eight years and one multi-grade teacher handling intermediate levels, Grade 4 to 6 with sixteen years of experience. Of the secondary mathematics teachers, two of them teaching Grades 7 and 8 with three and eight years of experience, one teaching Grades 8 and 9 with three years of experience, two teaching grade 9 and 10 with five and two years of experience, and one teaching Grade 10 with three years of experience.

Instruments

The primary sources of data of this present study were the list of learning contents in fraction extracted from the existing Curriculum Guide in Mathematics (Appendix C) and the transcriptions of the conducted two sessions of FGD (Appendix D). The List of Learning Contents in Fraction was categorized by grade level. This list was then subjected to content analysis taking for consideration the pattern that can be identified to describe how learning contents in fraction transcend from elementary to secondary levels.

On the other hand, FGD transcriptions reflected the one-hour FGD proceeding done on February 23, 2020 for the secondary, and March 2, 2020 for the elementary group at Magallanes, Sorsogon City. The first session was composed of six secondary teachers and the second session was composed of seven elementary teachers. Both sessions were audio-recorded.

The first FGD yielded a 19-page transcription that actually ran for 55 minutes while the second session had 12 pages that actually ran for 43 minutes. The transcription reflected the responses of the participants in English, Filipino and the native vernacular, Sorsoganon as spoken in Magallanes, Sorsogon. From these transcriptions that the data to the different research questions were deduced, categorized and interpreted.

RESULTS AND DISCUSSION

Transcending of learning contents in fraction from elementary to secondary level

From the collected qualitative data relevant to achieve the first goal of this study, the categorical themes were extracted. Using the Directed Content Analysis to answer this inquiry was guided by the theory of Jerome Bruner in 1966 known as the Concrete-Pictorial-Abstract Approach (CPA Approach henceforth). From this point, this study established the transcending of learning contents in fraction from elementary to secondary level as the sequence of instruction, namely: 1) Concrete and action-based representation using manipulative skills; 2) Pictorial representations of concrete objects; and 3) Abstract Notations using Mathematical Symbols.

Phase 1: Concrete and action-based representation using manipulative skills

Teachers shared different practices that they do inside the classroom where concrete and action-based representation using manipulative skills can be done especially in the early years of school. During the Concrete Phase, physical objects are used to solve math problems or plainly to make learners understand a math concept. This phase is dominantly applied in early numeracy years such as Grade 1 to Grade 3.

In Grade 1 Mathematics class, Teacher C can be observed to have plenty of objects used for manipulative skills. Other than the pictures mounted on walls of the classroom, Teacher C's classroom is filled with puzzles, figured boxes into 3D shapes using box, wood or plastic

materials, toys, miniature objects, and others. These instructional materials are used to develop manipulative skills of the learners at the early stage of learning, Grade 1. They are concrete objects that learners known to exist and these are clear and very familiar to them. Such objects can be used as springboard whenever teacher would like to make them understand a new concept in fraction.

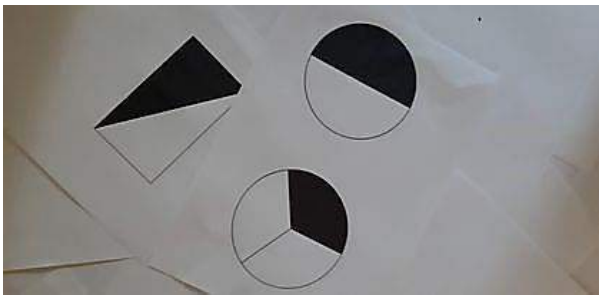
Picture 1. Samples of Manipulative Objects that Teachers Use during Concrete Phase

During the classroom observation conducted, Teacher C gave an example of how she demonstrated abstract concept of ½ using a round pizza pie. With a round pizza pie picture used as concrete object, Teacher C said that round pizza pies were familiar to almost all her learners thus potential to be used as springboard to understand the concept of ½. With this, she carefully explained that the pizza pie can represent one whole idea. Then, she carefully sliced one whole of pizza pie to show that one whole can be cut into two parts which one part can represent the idea of ½. She repeatedly did this strategy with other objects available in her classroom such as with different fruit objects, cutting them into two parts to mean the idea of ½.

Picture 2. The Concrete Object Examples of the Teacher during Class

For Teacher A, a Grade 3 teacher, her classroom was also equipped with concrete objects that can be used for different instructional purposes. For her, an empty classroom has lesser learning opportunities. In Picture 3, it showed the drawings of shapes that she did to emphasize the different fractions being taught.

Teacher A introduced the fraction concept by giving pupils circles and other shapes with shaded parts so everyone might be able to name the equivalent fraction. This concrete object is shown in Picture 3.



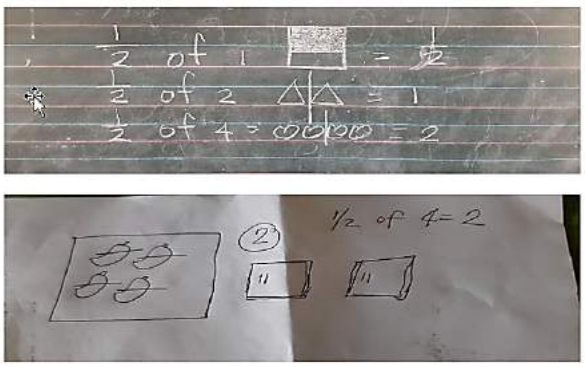
Picture 3. Drawn Concrete Object to Show Different Fractions

This picture illustrates how fractions may be taught to Grade 3. These drawn objects were cut so that learners can be able to touch and manipulate them. Like simple puzzles, learners can put them together to create the different concepts represented by the fraction. In this case, even the objects are drawn, they are cut out into concrete shapes so that learners can hold them, point them out whenever needed. Such manner may provide learners with avenues to utilize their senses to learn. Using senses especially in the early years of numeracy development is so essential and was proven to be one of

the best manners for early learners to learn better. Such approach is called Multisensory Approach where learning through the use of different senses helps students learn material in a more concrete manner (Learn Through Experience, 2018). Studies show that only certain parts of the brain activate during learning; therefore, visual learning will activate a different part of the brain than would olfactory learning, for example. It was cited in this blog that in a report by D.G. Treichler, he stated that "People generally remember 10% of what they read, 20% of what they hear, 30% of what they see, and 50% of what they see and hear." Combining the senses, therefore, is of benefit to students of all learning styles. In the case of transcending learning contents in Mathematics, it is clear from this first phase that making learners understand Mathematics concepts is basically through the use of manipulative and concrete objects such as the ones presented in the pictures.

Phase 2. Pictorial representations of concrete objects

After the Phase 1 or the concrete and action-based representation using manipulative skills in transcending learning contents in fractions was the Phase 2 or the pictorial representations of concrete objects. In Phase 2, teachers used different pictures to represent a concrete idea in preparation for a more sophisticated thinking to understand concepts in fraction. In Grade 1, the concept of ½ and ¼ was the basic content being taught for fraction. Picture 4 shows a sample material being used in this phase by Teacher A. The first illustration was used by Teacher D to introduce the concept of ½ and ¼. She used chalk and board to give a pictorial representation of the two fractions using different shapes and figures. The drawing of objects was once presented using real objects but this time, they were presented through a mere drawing on the board. The second illustration was used by Teacher B who was also teaching Grade 1. It was an illustration during a follow interview session since classroom observation was no longer possible to date.



Picture 4. Sample of Pictorial Representation of Fraction Concepts

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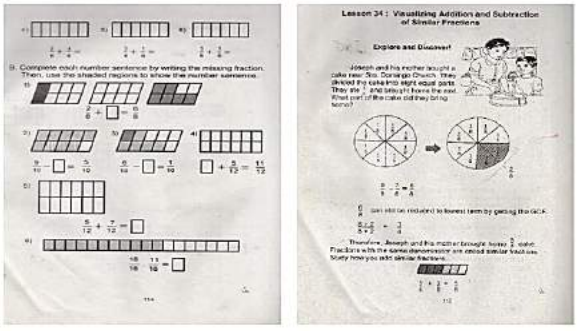
In the first picture of Picture 4, it can be seen that after writing the numerical symbol for ½, Teacher A followed it with drawings of different shapes. This is to give learners the idea of what is ½ when applied to concrete objects which was done earlier in the lesson. Opposite the illustration is the numerical concept of ½ that applies to the number or kind of objects being taken as a sample. This is also true to Teacher B who showed how she drew sample of concrete objects and associate them with the concept of ½ and ¼ in Grade 1 lessons for fraction. In similar instance, Teacher D who is also a Grade 1 teacher, exemplified how she taught lessons of ½ and ¼ through the use of manila paper. She asked learners to draw the different shapes themselves to see and assess how far learners understood the taught concepts of ½ and ¼.

Picture 5 showed a pictorial representation of fraction concepts from the textbook. Teacher E showed the researcher some of the fraction lessons in the textbooks with full illustrations of the concepts. She always used the textbooks since there are enough books for learners to use. The textbooks in Mathematics contained lessons in fractions and were well illustrated similar on Picture 5. In these pictures, there were enough illustrations to picture out the different concepts of each fraction. They came in different shapes and illustrations to gain the attention of the learners.



Picture 5. Sample Illustrations of Fractions Using Pictorial Representation from the Textbooks

Irons (2021) disclosed that children benefit greatly from the use of concrete materials and visual images when learning fractions. He also added that Jean Piaget description of stages of cognitive development where children assimilate new ideas emphasized the use of material and pictures as they interact within their environment. Likewise, Jerome Bruner also wrote about the importance of children interacting with their environment to promote learning. He described three broad stages of progression that were observed for all children –*enactive*, *iconic*, and *symbolic*. While all stages involved the use of materials in some way, the iconic stage focuses on 3D objects and 2D pictures. Irons further emphasized the great benefit from the use of concrete materials and pictures when learning fractions which have many aspects that need to be explored and then linked together to make sense of them.

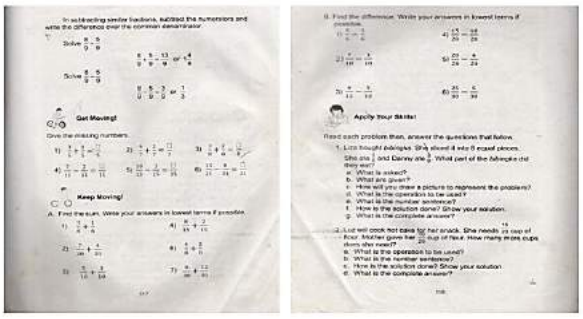
Further, Irons also warned the use of visual especially fraction models which do not automatically guarantee success. Care needs to be taken with selecting an appropriate model based on the prior experiences of the students with fractions. It is also important to consider the fraction content that is being taught and use the model that will clearly display the desired ideas.

Phase 3. Abstract Notations using Mathematical Symbols.

The last sequence to show the transcending of learning content in fraction was the Phase 3 or Abstract Notations using Mathematical Symbols. At this phase, learners are expected to gain substantial and adequate knowledge skills gained from the previous phases of Concrete and action-based representation using manipulative skills, and Pictorial representations of concrete objects. Without such foundation, learners may find difficulty in understanding the sophisticated knowledge and skills in fractions.

Such an idea is very important to note especially that a learner who solves mathematical problems at an abstract level does so without the use concrete objects or without drawing pictures. Understanding math concepts and performing math skills at the abstract level requires students to do this with numbers and math symbols only. Abstract understanding is often referred to as, "doing math in your head." Completing math problems where math problems are written and students solve these problems using paper and pencil is a common example of abstract level problem solving.

In Picture 6, different fractions in numerical representations and worded problems are displayed instead of pictures and illustration. These samples are used with Grade 4 learners. At this stage, learners are expected to gain basic knowledge of the fractions during their previous lessons and grades in mathematics. They have gained already the knowledge based from the Phase 1 and Phase 2 of instructional sequence in Mathematics instruction.



Picture 6. Sample Fraction Activities that Show Abstract Phase

These samples are taken from textbooks. In the textbooks, it can be observed that in the first phase of the lesson discussions, drawings and illustrations are still utilized to make the concepts clearer for the learners to understand. While the lesson is proceeding from motivation and discussion to evaluation phase, it can be observed that presentations also transform from illustrated to abstract ideas in the form of numerical and worded forms.

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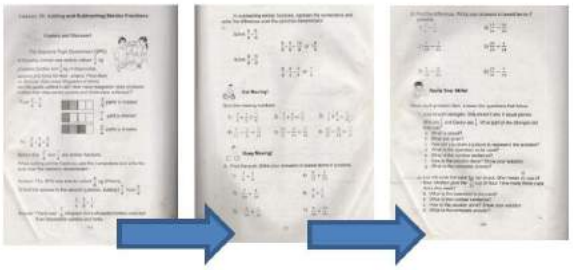
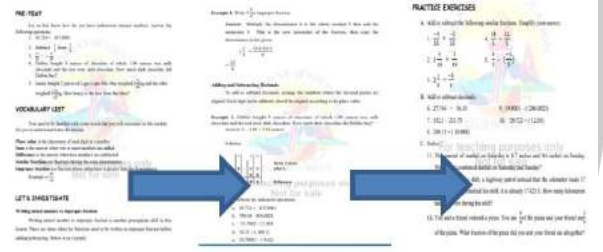


Picture 6. Sample from Textbooks that Observe the Sequence of the Transcend of Learning Contents

The movement of the lesson sequence can be observed in the horizontal progression and vertical articulation integrated in the principles of spiral progression approach. Specifically, in teaching and learning fraction, vertical articulation of the lesson is noted with what

Picture 6 is exemplifying. In one content in fraction, the Concrete-Pictorial-Abstract Approach by Jerome Bruner is evident. Likewise, horizontal progression can be observed in the sequence of learning contents from one grade level to another. In doing so, the CPA Approach is also evident. In Grade 1, most of the lessons in fraction are presented through manipulative or concrete objects even in the vertical articulation because this stage demanded instruction to use concrete objects for the learners even if the lesson is in the discussion, practice and evaluation phase. While the grade level progresses, CPA Approach is also evident where there is less utilization of manipulative and concrete object, only pictorial representation and abstraction have become dominant practices.

In the secondary level, Picture 7 indicates how a fraction lesson transcends from discussion to practice to evaluation. Since in the secondary level, there are few lessons on fractions, materials are only limited to Grade 7 Mathematics modules only. At this level, it can be observed from the pictures that few illustrations are employed and less manipulative; there are more numerical representations instead. These samples were lessons from Operations on Rational Numbers.



Picture 8. Sample from Textbooks that Observe the Sequence of the Transcend of Learning Contents in Grade 7

Furthermore, fraction lessons in higher grade levels in the secondary are integrated and implied in the different lesson contents in Mathematics. Fractions can be in percentage and decimal forms. For instance, in Grade 10, fraction lessons are used in the Statistics and Probability such as 50% or 0.50 of the total number of population. In Grade 8, fractions come in rational numbers such as the equivalent of ½ is used as 0.50 or ¼ as 0.25. In Grade 9, fractions are presented in radical forms such ½ is now $\sqrt{2}$ or $2^{\frac{1}{2}}$.

Teachers need to be skillful in lesson transition beyond modeling, being proactive, reflective, and writing about all relevant math aspects (Loughran, 2009). Emphasizing the effective use of manipulatives, teachers can explicitly connect abstract math concepts and manipulatives to establish a socio-mathematical routine for the advantage of understanding mathematics and promoting transfer of learning (Kamina and Iver, 2009).

The CPA Approach is clearly seen may it be in the movement of learning content via horizontal progression and vertical approach within the principles of spiral progression. Based from the investigation conducted in this present study, the CPA Approach is the encompassing mathematics learning content from classroom instruction within and every after the grade level. The purpose of teaching through a concrete-to-representational-to-abstract sequence of instruction is to ensure students truly have a thorough understanding of the math concepts/skills they are learning. When students who have math learning problems are allowed to first develop a concrete understanding of the math concept/skill, then they are much more likely to perform that math skill and truly understand math concepts at the abstract level.

Scaffolding the Learning Contents in Fraction that Transcend from Elementary to Secondary Level

The strategies that were gathered are discussed at this juncture are categorized. There were different strategies that teachers employed in applying the principles of CPA Approach in their mathematics instruction especially in fraction. They are discussed correspondingly.

Conduct of diagnostic assessment among learners. The first activity that teachers did was to conduct a pre-test or any means of diagnostic activity among their learners especially at the first phase of the lesson or school year. The results of the diagnostic assessments help teachers to decide the level of the learning content that they will have to do as well as identify where to begin their instruction.

A secondary teacher shared that difficulties of the learners were discovered based on the result of any assessment (Lines 882-882). Diagnostic assessment is very important particularly during the first day of the school year as a means to familiarize with the abilities that new set of learners acquire. At this point the teacher can decide what instructional adjustments that she can do so that her students can run after the lesson.

Most of the time, the difficulties in fraction is observed in one class. Learners belonging to one class share common difficulties. However, there were instances when the teacher can observe exceptional abilities or difficulties that learners experience. With this, it is when

the teacher decides for appropriate intervention to employ with the class.

Review of learner's previous knowledge. Another strategy that teachers apply when scaffolding learning contents was to review learner's previous knowledge. This is an established practice among teachers. This strategy is even applied to every lesson conducted in the class. Before starting a new lesson, teachers conduct review of the learner's previous knowledge because this previous knowledge will help learners in understanding the new concepts to teach. Teachers use pen and paper test, question and answer portion, and unlocking of difficulties for some quick checking of their learner's previous knowledge (Lines 817-819).

The essence of the CPA Approach is to build foundational skills for the learners to have adequate skills to learn a particular concept in fraction. With the use of this strategy, the review of learner's previous knowledge, teachers can determine some instructional adjustments so that she can establish the needed foundational skill to proceed to the new concepts successfully or immediately proceed to the new concept whenever applicable.

An elementary teacher shared that it essential to determine the learners' previous knowledge especially with elementary learners such as Grades 1 to 3 (Lines 245-248). These years are very crucial for them because this is the beginning of their academic journey. Teachers handling the primary years ensured that the basic skills stated in the curriculum guide are acquire by the learner before they can proceed to the next grade level. They do remediation and other types of intervention that they think are appropriate to apply among the identified learners with difficulties.

Use of real, touchable, manipulative and concrete objects. This strategy is very common among primary grades teachers. Teachers use manipulative objects such as puzzles and cut out shapes. Their classrooms are almost filled with displayed and mounted objects that make their classroom colorful and attractive to children in the first place. Their walls are also full of pictures and drawings that are useful and purposeful in the instruction. This strategy is essential especially with the first phase of the CPA Approach. With these concrete objects, learners can relate their experiences to the concepts that their teachers are about to teach them.

During the course of the discussion in fraction, Teacher D usually used paper cut outs that learners can manipulate to show how they view ½ or ¼; if the concept in the mind of the learners confirmed to what the Teacher D has taught. This is usually done during evaluation or application phase of the lesson. Similarly, as what was discussed in the previous result of the study, illustrations of pizza pie and watermelon were used as manipulative and concrete objects that can help learners relate in the concepts being taught. Manipulating concrete objects may help students understand that fractions were applicable to real life (Hafiziani 2015). Learners immediately feel the benefits of learning fractions to solve real life problems.

Peer Support or Tutorial Approach Learning. One of the FGD participants shared that in the secondary levels, learners are given the chance to be taught by their classmates and peers (Lines 855-860). They assigned learners who are advanced and knowledgeable about the topics they are teaching to be a para-teacher to their classmates. They did the tutorial sessions after classes, that is, 4:00 PM – 5:00 PM. Another participant shared that he grouped learners according to the result of their assessment to be able to identify who will become the para-teacher and who will be the learners to be taught by them (Lines 866 to 878).

This strategy may help teachers in teaching learners who did not have a good grasp of the lesson presented. They were helped by the learners who are more knowledgeable and were identified by the teacher. Such manner lets the learners become more comfortable to learn since it is their friends who teach them. They will not hesitate to ask whatever is confusing them. They will not be shy to clarify things because they are friends. However, there were instances when there were learners who would not admit that they were being taught by their peers instead of their teachers (Lines 887-892). This complicates the situation especially teacher's intentions are misinterpreted by the learners.

Learner-Made Learning Materials. Teachers engaged learners to craft their own learning materials especially with the high school learners. The aim is to check how far the learners have gone in terms of their learning in fraction out of the material that they made. There were teachers who shared that they required their learners to submit portfolio where they track learners' progress in mathematics in the different content areas and one of these was fraction. In the case of Grade 8 to 10 where fraction lessons are integrative in the other content in Mathematics, teachers can already evaluate the extent of their learning through their outputs and quality of responses in the activities included in the portfolio.

In elementary level, learner-made materials are required at the end of every quarter in the form of projects or outputs. In Mathematics, they required learners to create a model of an object where they can exemplify the idea of ½ and ¼ (for Grade 1). In this ways, teachers may be confident about their learners being familiar with the different portions of fraction in an object. A teacher shared that such material can be easily understood by the learners of Grade 1 especially that they can concretely see the object and the abstract idea of the fraction in an object.

Contextualizing and Localizing Learning Modules. Contextualizing and localizing the learning modules are initiatives that teachers themselves will have to do with the learning materials particularly when these materials come from DepEd Central Office. The original developer of the materials may have come from other places, using different languages and sets of culture which may have different impact to the learner. The teacher usually identifies different areas of the module that must be contextualized and localized which later they would revise and fit into the culture and background of the learners. Another way to apply contextualization and localization is to simplify lessons especially for the slow learners.

This way, teachers may choose the design that they would like to place in the module considering the culture and context of their learners. Teachers of the learners are the ones who knew their learner more in terms of socio-economic and academic profile so the designs of their learning material have to be aligned with their learners. Because of the principles of contextualization and localization, teachers tend to make or create their own learning materials tailored-fit for their learners.

Providing Learning Support Facilities. In a classroom of elementary learners, teachers provided different learning facilities to maximize the learning avenue to every learner. This is usually done according to content area. For Mathematics, teachers build Math Corner in one area of the classroom where relevant manipulatives are placed for the learners. There are posted and mounted charts and pictures on the wall which are attractive to learners so that they can be motivated to learn.

Teachers strived to make their classroom a small area yet has plenty of avenues to learn even up to the point of spending their own money and soliciting from parents to help teachers in the expenses incurred during the refurbishing of the classrooms to make them suitable and conducive to learning.

Use of innovative teaching strategies may help learners overcome their fear and misconception that they may have about fractions (Purwadi, Sudiarta & Suparta, 2019).Using the CPA strategy has a positive effect on students' mathematical conceptual understanding and students' mathematical represention on fractions. This result is consistent with Sarfo et al. (2014) which stated that the CPA strategy had a positive effect on students' conceptual understanding of geometry and algebra (Flores, 2010). Witzel et al. (2003) also revealed that the CPA strategy was effective for learning mathematics for students with or without a learning disability. Learning fractions using CPA strategy started by manipulating concrete objects, and using a variety of representations of fractions. This strategy helps to feel less anxious when dealing with the abstract concept of fractions. CPA strategy also correlates with the level of the cognitive development of elementary school students. However, it does not rule out the possibility that CPA strategy is effective middle and high school students (Hafiziani, 2015).

CONCLUSION AND RECOMMENDATION

The learning contents in fraction transcend from elementary to secondary level following the framework of the CPA Approach with the sequence, namely: 1) Concrete and action-based representation using manipulative skills; 2) Pictorial representations of concrete objects; and 3) Abstract Notations using Mathematical Symbols. The learning contents in fraction scaffold using strategies, namely: 1) conduct of diagnostic assessment among learners; 2) review of learner's previous knowledge; 3) use of real, touchable, manipulative and concrete objects; 3) Peer support or tutorial approach learning; 4) learner-made learning materials; 5) contextualizing and localizing learning modules; and 6) providing learning support facilities.

The learning contents in fraction transcend from elementary to secondary level according to the framework and principles of CPA Approach. There are varied strategies that teachers use to scaffold the learning content in fraction that transcend from elementary to secondary level.

Elementary and Secondary teachers may become attentive and observant about they principles of their teaching to ensure that different empirical approaches such as the CPA Appoach may appropriately and accordingly apply in their instructional practices especially when teaching fraction to fulfill the needed

skills of learners in preparation to the next level of learning contents in fraction and other mathematical concepts. Varied and purposeful scaffolding strategies provided in the proposed handbook may be utilized to apply CPA in teaching fraction. The proposed teacher's handbook entitled Concrete-Pictorial-Absract Approach (CPA Approach) to Teach Fraction & Other Mathematical Concepts – A Teacher's Handbook may be utilizing to enhance the teaching skills of mathematics teachers in elementary and secondary. Action research along the utilization of CPA Approach as intervention may be conducted.

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